COLLOQUIUM 2020-2021

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Black holes and Acoustic black holes.
Spectral Analysis of a Rod with a Sharp End.
Does a Black Hole Make Sound?

Part I: Preliminaries

September 18 2020

Abstract. **Motivation**: The flow of engineering papers that describe the so-called acoustic black hole, e.g. a beam with a monotonically $\downarrow$ thickness toward one of the end pts.

Unusual Phenomena:
(a) The time of propag-n of a signal toward such an endpoint is infinite so that the signal is "trapped."
(b) The amplitude of a signal increases with no bound near this point.

Hypothesis:
(c) The reflection coeffcient from the shrap end is zero.

Objective:
Rigorous study of the spectral properties of the St.-L. prb-m for a second-order diff.operator on a finite interval with the coeff-s vanishing at one of the end pts.
Physical meaning: Study a rod instead of a beam.

Results:
(1) Clafrication of Phenomena (a), (b) above.
(2) Description of the class of the coeff-s of the diff.operator for which the essential spectrum appears.
(3) Study of the location of the essential spectrum.
(4) Study of the absence of the discrete spectrum.
(5) Study of the specral properties for two models, almost sharp end & sharp end, and their connection.

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COLLOQUIUM 2020-2021

Qiao Zhuang

Immersed finite element methods for second order hyperbolic equations in inhomogeneous media

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September 25 2020

Abstract. In this presentation, we reanalyze the fully discrete Partially Penalized Immersed Finite Element (PPIFE) method. Utilizing the error bounds given recently for elliptic interface problems, we are able to derive optimal a-priori error bounds for this PPIFE method not only in the energy norm but also in L^2 norm under the standard piecewise H^2 regularity assumption in the space variable of the exact solution, rather than the excessive piecewise H^3 regularity. Numerical simulations for standing and travelling waves are presented, which corroborate the reported error analysis.

COLLOQUIUM 2020-2021

William R. Green

Dispersive estimates for the Dirac equation

Department of Mathematics

Rose-Hulman Institute of Technology

November 13 2020

Abstract. The Dirac equation was derived by Dirac in 1928 to model the behavior of subatomic particles moving at relativistic speeds. Dirac formulated the following hyberbolic system of partial differential equations

$$i\psi_t(x,t) = (D_m + V(x))\psi(x,t), \qquad \psi(x,0) = \psi_0(x).$$

Here $D_m = -i\alpha \cdot \nabla + m\beta$ is a matrix-valued differential operator, $\psi(x,t)$ is vector-valued, $V(x)$ is an external potential, and $m \geqslant 0$ is the mass. The matrices composing D_m satisfy certain anti-commutation relationships which yield the identity

$$D_m^2 = -\Delta + m^2.$$

This allows one to interpret the Dirac equation as a sort of square root of a system of Klein-Gordon or wave equations when $m > 0$ and $m = 0$ respectively.

The Dirac equation is considerably less well studied than other dispersive equations such as the Schrödinger, wave or Klein-Gordon equations. We will survey recent work on time-decay estimates for the solution operator in dimensions $n \leqslant 3$. Specifically the mapping properties of $e^{-itH}P_c(H)$ where $H = D_m + V$ and P_c is the projection onto the absolutely continuous spectral subspace associated to H. As in other dispersive equations, the existence of threshold eigenvalues and resonances affect the dynamics of the solution. The talk will survey joint works with B. Erdogan (Illinois), M. Goldberg (Cincinnati) and E. Toprak (Rutgers).

COLLOQUIUM 2020-2021

Libin Rong

Recent developments in modeling HIV infection and treatment

Department of Mathematics

University of Florida

October 30 2020

Abstract. HIV infection is still a serious public health problem in the world. Highly active antiretroviral therapy can suppress viral replication to a very low level but cannot eradicate the virus. Mathematical models, combined with experimental data, have provided important insights into HIV dynamics, immune responses, and drug treatment. However, there are still a lot of questions that remain unanswered, for example, are some drugs more effective than others? whether treatment intensification brings benefits to patients? what causes multiple infection that may lead to drug resistance and immune escape? are there any other sources, besides the latent infection, that contribute to HIV persistence despite long-term therapy? In this talk, I will present our recent modeling efforts in addressing these issues and also discuss their implications for the management of HIV infection.

COLLOQUIUM 2020-2021

William Green

Dispersive Estimates for the Dirac Equation

Department of Mathematics,

Rose-Hulman Institute of Technology

November 13 2020

Abstract. The Dirac equation was derived by Dirac in 1928 to model the behavior of subatomic particles moving at relativistic speeds. Dirac formulated the following hyperbolic system of partial differential equations $i\psi_t(x,t) = (D_m + V(x))\psi(x,t),\ \ \psi(x,0) = \psi_0(x)$. Here $D_m = -i\alpha\cdot\nabla + m\beta$ is a matrix-valued differential operator, $\psi(x,t)$ is vector-valued, $V(x)$ is an external potential, and $m \geqslant 0$ is the mass. The matrices composing D_m satisfy certain anti-commutation relationships which yield the identity $D_m^2 = -\Delta + m^2$. This allows one to interpret the Dirac equation as a sort of square root of a system of Klein-Gordon or wave equations when $m > 0$ and $m = 0$ respectively. The Dirac equation is considerably less well studied than other dispersive equations such as the Schrodinger, wave or Klein–Gordon equations. We will survey recent work on time-decay estimates for the solution operator in dimensions $n \leqslant 3$. Specifically the mapping properties of $e^{-itH}P_c(H)$ where $H = D_m + V$ and P_c is the projection onto the absolutely continuous spectral subspace associated to H. As in other dispersive equations, the existence of threshold eigenvalues and resonances affect the dynamics of the solution. The talk will survey joint works with B. Erdogan (Illinois), M. Goldberg (Cincinnati) and E. Toprak (Rutgers).

COLLOQUIUM 2020-2021

Hassan Mohamed Abdelalim Abdalla

Dynamic optimization in structural mechanics

Polytechnic Department of Engineering & Architecture,

University of Udine, Italy

January 29 2021

Abstract. The application of optimization theory in engineering allows one to choose, among several feasible designs, the one which is optimal with respect to a given criterion, complying with prefixed constraints. Based on the nature of design variables, optimization problems can be classified into two broad categories. In the former, values of design parameters that make a prescribed function minimum and subject to certain constraints are sought. Such problems are called static optimization problems. In the second category, the design variables are themselves continuous functions that minimize an objective functional subject to a set of constraints, typically in integro-differential form. Such problems are called dynamic optimization problems. As far as dynamic problems are concerned, Pontryagin's Principle can be used to derive necessary conditions for optimal solutions, yet rarely solvable analytically, hindering one to resort to numerical approaches. One of the strategies is the application of the so-called pseudospectral methods, which mainly transcribe a dynamic (infinite-dimensional) problem into a static (finite-dimensional) one. In this talk, theoretical preliminaries of dynamic optimization theory are given, hints on pseudospectral methods are presented and application to a few problems in structural mechanics are addressed.

COLLOQUIUM 2020-2021

Lakmali Weerasena

Department of Mathematics

The University of Tennessee at Chattanooga

Design of an algorithm with multiple-cost efficient rules for the generalized multi-objective set cover problem

March 12 2021

Abstract. Set covering optimization problems (SCPs) are relevant and of broad interest since their extensive applications in the real world. This study addresses the generalized multi-objective SCP (GMOSCP), which is an augmentation to the well-known multi-objective SCP (MOSCP) problem. A mathematically driven heuristic algorithm is proposed based on a branching approach of the feasible region to approximate the Pareto set of the GMOSCP. The algorithm consists of a number of components including an initial stage, an constructive stage, as well as an improvement stage. Each of these stages contributes significantly to the performance of the algorithm. In the initial stage, we use an achievement scalarization approach to scalarize the objective vector of the GMOSCP, which uses a reference point and a combination of weighted l_1 and l_∞ norms of the objective function vector. Uniformly distributed weight vectors defined with respect to this reference point support the constructive stage to produce a more widely and uniformly distributed Pareto set approximation. The constructive stage identifies feasible solutions to the problem based on a Lexicographic set of selection rules. The improvement stage reduces the total cost of selected feasible solutions, which benefits converging of the approximations. We propose multiple cost-efficient rules in the constructive stage and investigate how they affect approximating the Pareto set. We have used a diverse set of GMOSCP instances with different parameter settings for the computational experiments.

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COLLOQUIUM 2020-2021

Jeremy Hale

Modeling additive manufacturing machine health
using witness parts and a partially observed Markov decision process

Department of Industrial and Systems Engineering,

University of Tennessee, Knoxville

March 19 2021

Abstract. Additive manufacturing is a production process that iteratively adds material to build a final product, as opposed to more conventional methods of subtractive for formative manufacturing. These additive processes allow the manufacture of highly complex designs with flexible production capabilities. However, these same features have made quality difficult to control and maintain for AM production, and the relatively complex nature of the machines difficult to implement traditional methods of monitoring. This research proposes a framework of witness parts, produced systematically, to monitor additive manufacturing machines' condition and health. This data can then be used to model machine health as a partially observed Markov decision process to predict, detect, and respond to machine health and quality changes. This model is proposed to be implemented more easily and cheaply than sensor-based, in situ, live monitoring systems. The end of the time will also feature a short presentation on data analysis in industrial engineering and research opportunities by research advisor and professor Dr. Mingzhou Jin.

COLLOQUIUM 2020-2021

Ziwei Ma

A stochastic Gompertz diffusion model for untreated human glioblastomas

Department of Mathematics,

University of Tennessee at Chattanooga

April 9 2021

Abstract. Gompertz curve, a sigmoid function which describes growth as being slowest at the start and end of a given time period, has been applied to several types of solid tumors. As a purely phenomenological growth curve, some tumors have a particular growth pattern that can be described by a Gompertz growth model. However, there are discrepancies between clinical or experimental data and theoretical predictions. These discrepancies may be due to internal environmental fluctuations and variation among patients. To consider such environmental fluctuations and individual patients, a stochastic Gompertz model is proposed for untreated human glioblastomas, primary malignant brain tumor. Using untreated human glioblastoma data collected in Trondheim, Norway, we first fit the average growth to a Gompertz curve, then find a white noise term for the growth rate variance. Combining these two parts, we obtain a new type of Gompertz diffusion dynamics, which is stochastic differential equation (SDE). Instead of growth curves predicted by deterministic models, our SDE model predicts a band with a center curve as the tumor size average and its width as the tumor size variance over time. We carry out numerical studies to predict the patient survival time with a prescribed probability. Also, our model can be applied to studies of tumor treatments. As a demonstration, we numerically investigate different protocols of surgical resection using our model and provide possible theoretical strategies.

COLLOQUIUM 2020-2021

Eleni Panagiotou

Knot polynomials and Vassiliev measures of open and closed curves in 3-space

Department of Mathematics,

University of Tennessee at Chattanooga

April 16 2021

Abstract. Knots and links appear in our everyday life, such as our shoelaces, or polymer melts, proteins and DNA. Measuring the complexity of such knots has been a challenge for many decades, due to the fact that they have open ends and thus do not satisfy the conditions for mathematical knots. Mathematical knots can be characterized using knot and link polynomials. In this talk we introduce a method to measure entanglement of curves in 3-space that extends the notion of knot and link polynomials to open curves in 3-space. We define the Jones polynomial of curves in 3-space and show that for open curves, it has real coefficients and it is a continuous function of the curve coordinates and for closed curves, it is a topological invariant, as the classical Jones polynomial. We show the Jones polynomial can attain a simpler expression for polygonal curves and we provide a finite form for their computation in the case of polygonal curves of 3 and 4 edges. As the complexity of the curves in 3-space increases, the computation of the Jones polynomial increases as well. We introduce the Vassiliev measures of entanglement of open curves. We will show that these capture some of the information of the Jones polynomial with much less computational cost.

COLLOQUIUM 2020-2021

Philip Smith

The Second Vassiliev measure of polygonal curves in 3-space

Student, Department of Mathematics,

University of Tennessee at Chattanooga

April 23 2021

Abstract. Open knots and links are present in many physical systems, such as polymers and biopolymers. These physical filaments can be represented by polygonal chains in 3-space, whose complexity can be studied by using tools from Knot Theory. A second Vassiliev invariant, the Casson invariant, is a measure of topological complexity which can distinguish knots and links, as knot and link polynomials do, but it is easier to calculate. Recently, the second Vassiliev measure was defined for open curves in 3-space. For open curves, this is a measure of higher order complexity that is a continuous function of the chain coordinates. We apply the Casson measure to polygonal curves of varying length to examine how their complexity varies as a function of length. Our results will be used to quantify entanglement complexity in proteins and other polymers.