THE UNIVERSITY OF TENNESSEE CHATTANOOGA

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# Steel heat treating: mathematical modelling and numerical simulation of an industrial problem

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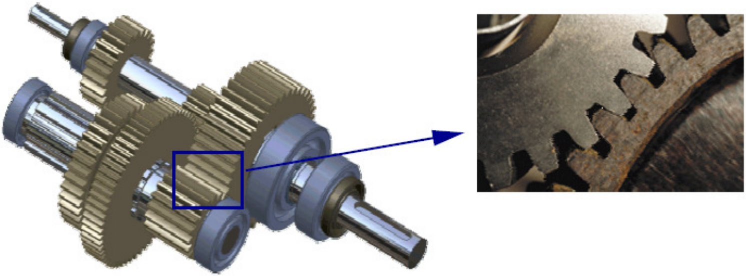
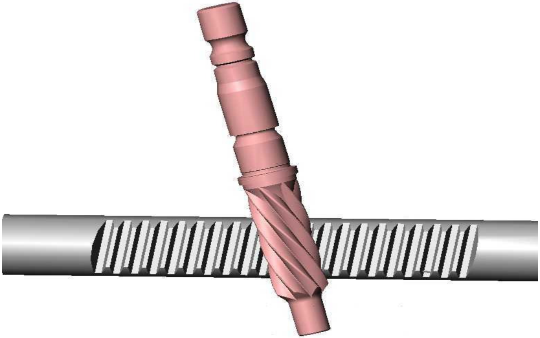
# Steel hardening

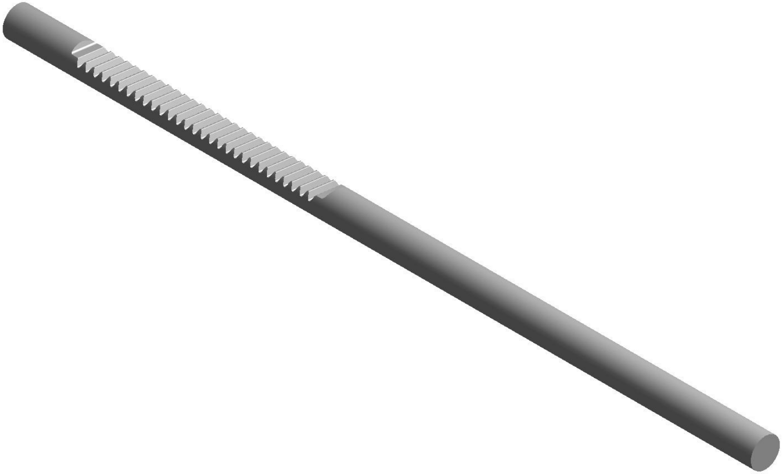
In the automotive industry, many important moving (rotating/translating) pieces are in close contact in order to transmit the desired rotation/translation movement: gear wheels, toothed rings, bevel gears, rack and pinion, etc.



One common example is the steering rack and pinion.

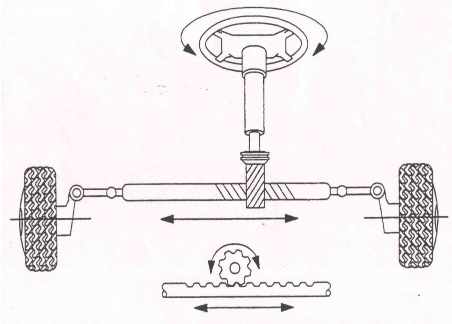


# Steel hardening



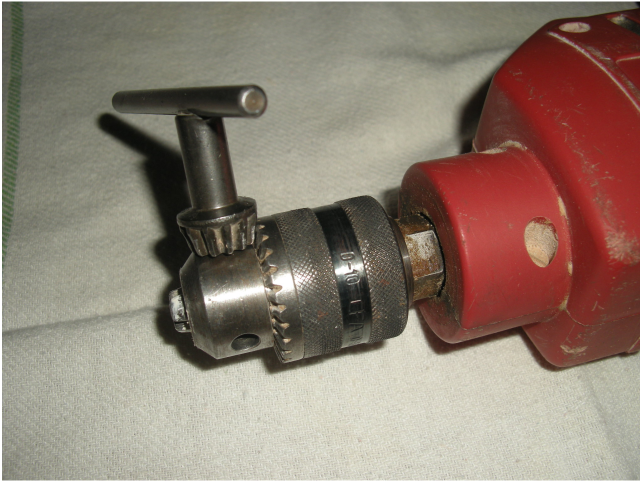
# Steel hardening

The rack and pinion are very important workpieces inside an automobile. It is part of the steering system of the vehicle. The lifetime of the rack and pinion kit must be of at least 20 years!



# Steel hardening

Wear and abrasion: drill chuck and key.



# Steel hardening

These workpieces are made of **steel**.

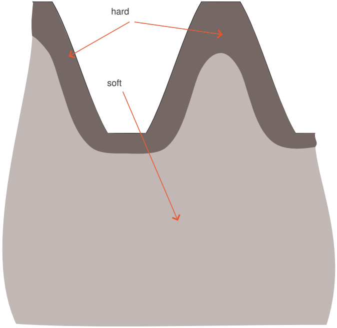
Prior to any hardening treatment, steel is a ductile material.

Rotating/translating workpieces in close contact are subject to stresses during its lifetime. **Hardening treatment** is necessary in order to avoid wear and abrasion.

# Steel hardening

A convenient hardening treatment is then applied in order to produce:

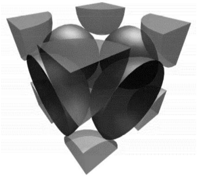
- a hard boundary layer to hinder wear and abrasion, and
- a soft inner part to reduce fatigue.



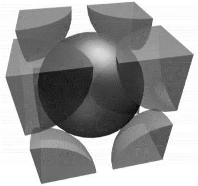
# Some facts on steel

Steel is an iron based alloy.

Iron may appear in two type of crystal lattices:



fcc

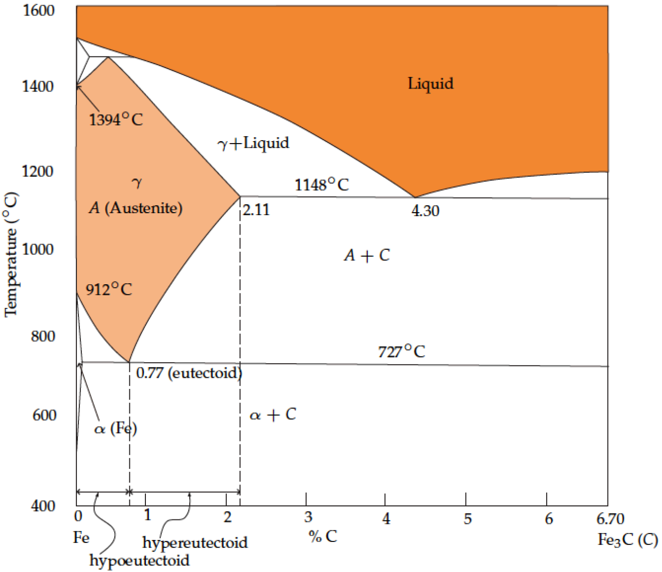


bcc

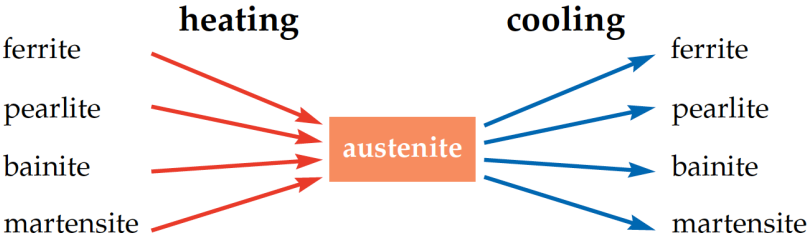
Different solid phases in steel:

- **Austenite**: Solution of C in fcc iron. Only possible if concentration of C up to 2.11%; if so, only possible at a high temperature range.
- **Ferrite**: Nearly pure bcc iron.
- **Pearlite**: Lamellar structure of ferrite and cementite ($Fe_3C$).
- **Martensite**: Tetragonally bbc iron crystal distorted by C atoms. It can only stem from austenite.

# Iron Carbide Phase Diagram



# Iron Carbide Phase Transitions




**Phase transitions in hypo/hyper/eutectoid steel**

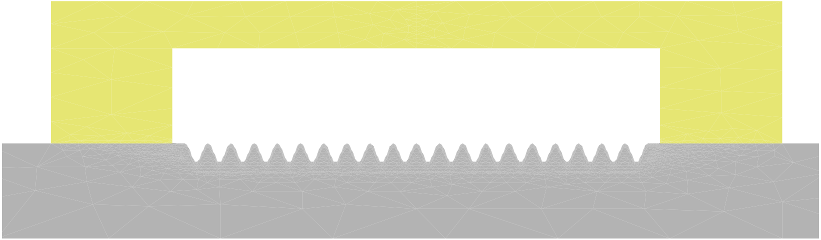
- Austenite → pearlite, bainite (slow cooling down temperature rate)
- Austenite → martensite (very rapid cooling down temperature rate)

**Phases have different physical properties**

- Pearlite: soft and ductile.
- Martensite: hard and brittle.

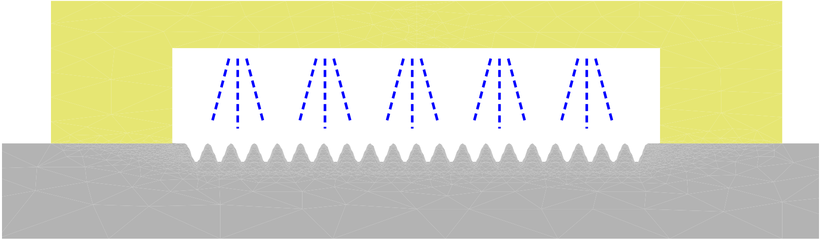
# The industrial procedure: 1. Heating

An **inductor** (copper) is put in contact with the workpiece.
A high frequency electric current (83 kH) is supplied. This induces eddy currents.
Joule's effect heats up the boundary layer till austenization is reached.

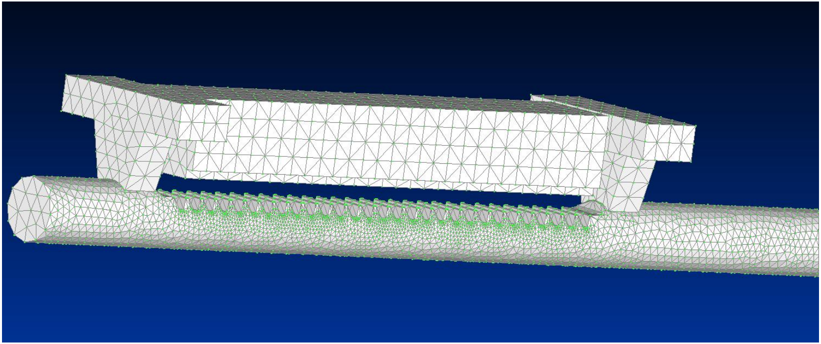


# The industrial procedure: 2. Cooling

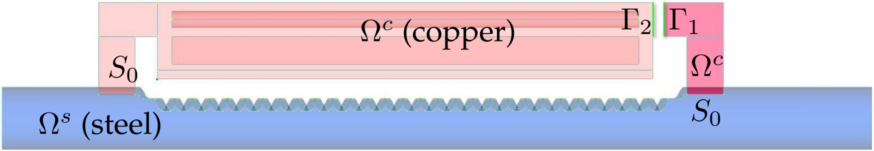
The power supply is switched off, and the workpiece is then **quenched** (aquaquenching).
**Martensite** is produced just where it is needed.



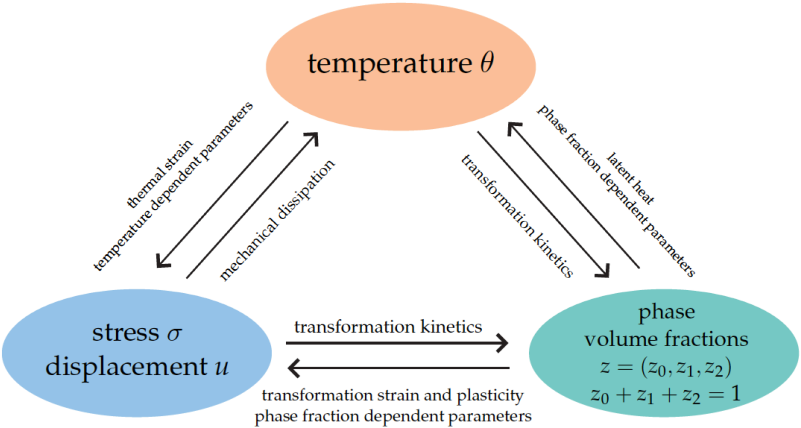
# The industrial procedure: 2. Cooling



# Rack and inductor



# Thermomechanical phenomena



# Steel hardening: Maxwell equations

# Steel hardening

# Steel hardening

This monument was unveiled on November 25th 2008, George Street, Edinburgh, Scotland, UK.

# Thermomechanical + electromagnetics modeling

D. Hömberg, W. Weiss, K. Chełminski, D. Kern (WIAS, Berlin) proposed the next model:

**Heating process** $t \in [0, T_{\mathrm{h}}]$

- Electromagnetics production: $\varphi$, $\boldsymbol{A}$;
- Viscoelasticity (quasi static model): $\boldsymbol{\sigma}, u$;
- Phase fractions (austenite): $a$;
- Temperature: $\theta$.

**Cooling process** $t \in [T_{\mathrm{h}}, T_{\mathrm{c}}]$

- Viscoelasticity (quasi static model): $\boldsymbol{\sigma}, u$;
- Phase fractions (austenite and martensite): $a, m$;
- Temperature: $\theta$.

# The heating process

**Electromagnetics production (main heat source):**
**Maxwell equations in harmonic regime**

$$
\begin{cases}
\nabla \cdot (b(\theta)\nabla\varphi) = 0 \text{ in } \Omega_{T_{\mathrm{h}}} = \Omega \times (0, T_{\mathrm{h}}), \\
\frac{\partial\varphi}{\partial n} = 0 \text{ on } (\partial\Omega \setminus \Gamma) \times (0, T_{\mathrm{h}}), \\
\varphi = 0 \text{ on } \Gamma_1 \times (0, T_{\mathrm{h}}), \quad \varphi = \varphi_0 \text{ on } \Gamma_2 \times (0, T_{\mathrm{h}}), \\
b_0(\theta) i\omega \boldsymbol{A} + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}\right) - \delta\nabla(\nabla \cdot \boldsymbol{A}) = -b_0(\theta)\nabla\varphi \text{ in } D \times (0, T_{\mathrm{h}}), \\
\boldsymbol{A} = 0 \text{ on } \partial D \times (0, T_{\mathrm{h}}),
\end{cases}
$$

$D$ is a big box containing the set of conductors (air+copper+steel),
$\Gamma = \Gamma_1 \cup \Gamma_2$,
$\omega = 2\pi f$ is the pulsation,
$f = 83$ KHz is the frequency.

# The heating process

**Viscoelasticity (quasi static model)**

$$\begin{cases} -\nabla \cdot \boldsymbol{\sigma} = F \text{ in } \Omega^{\mathrm{s}} \times (0, T_{\mathrm{h}}), \\ \boldsymbol{\sigma} = \boldsymbol{K}\Big(\boldsymbol{\varepsilon}(u) - A_1(a,m,\theta)\boldsymbol{I} - \int_0^t \gamma(a,m,a_t,m_t,\theta)\boldsymbol{S}\,\mathrm{d}\tau\Big), \\ u = 0 \text{ on } \Gamma_0 \times (0, T_{\mathrm{h}}), \\ \boldsymbol{\sigma} \cdot n = 0 \text{ on } (\partial\Omega^{\mathrm{s}} \setminus \Gamma_0) \times (0, T_{\mathrm{h}}), \end{cases}$$

where $\boldsymbol{K} = \boldsymbol{K}_{ijkl}$ is the stiffness tensor, namely

$$\boldsymbol{K}_{ijkl} = \bar{\lambda}\delta_{ij}\delta_{kl} + \bar{\mu}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \text{ for all } i,j,k,l \in \{1,2,3\}$$

where $\bar{\lambda} \geq 0$ and $\bar{\mu} > 0$ are the Lamé coefficients of steel;
$\boldsymbol{\varepsilon}(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ is the strain tensor;
$\int_0^t \gamma(a,m,a_t,m_t,\theta)\boldsymbol{S}\,\mathrm{d}\tau$ gives the model, through the function $\gamma$, of the transformation induced plasticity strain tensor, where $\boldsymbol{S} = \boldsymbol{\sigma} - \frac{1}{3}\operatorname{tr}\sigma\boldsymbol{I}$ is the deviator of $\sigma$.

$$A_1(a,m,\theta) = q_a a(\theta - \theta_a) + q_m m(\theta - \theta_m) + q_r(1-a-m)(\theta - \theta_r),$$

**Austenite**

$$\begin{cases} a_t = \dfrac{1}{\tau_a(\theta)}(a_{\mathrm{eq}}(\theta) - a)\mathscr{H}(\theta - A_s) \text{ in } \Omega^{\mathrm{s}} \times (0, T_{\mathrm{h}}), \\ a(0) = 0 \text{ in } \Omega^{\mathrm{s}}, \end{cases}$$

**Temperature**

$$\begin{cases} \alpha(\theta, a, m, \boldsymbol{\sigma})\theta_t - \nabla \cdot (\kappa(\theta)\nabla\theta) \\ \qquad +3\bar{\kappa}q(a,m)\theta\left(\nabla \cdot u_t - 3A_2(a_t, m_t, \theta)\right) \\ \quad = \frac{1}{2}b(\theta)|i\omega\boldsymbol{A} + \nabla\varphi|^2 - \rho L_a a_t \\ \qquad +A_2(a_t, m_t, \theta)\,\mathrm{tr}\,\boldsymbol{\sigma} + \gamma(a, m, a_t, m_t, \theta)|\boldsymbol{S}|^2 \text{ in } \Omega_{T_{\mathrm{h}}}, \\ \dfrac{\partial\theta}{\partial n} = 0 \text{ on } \partial\Omega \times (0, T_{\mathrm{h}}), \\ \theta(0) = \theta_0 \text{ in } \Omega. \end{cases}$$

$$\alpha(\theta, a, m, \boldsymbol{\sigma}) = \rho c_\varepsilon - 9\bar{\kappa}q(a,m)^2\theta - q(a,m)\,\mathrm{tr}\,\boldsymbol{\sigma},\ \bar{\kappa} = (3\bar{\lambda} + 2\bar{\mu})/3$$

$$q(a,m) = q_a a + q_m m + (1 - a - m)q_r;$$

$$A_2(a_t, m_t, \theta) = q_a a_t(\theta - \theta_a) + q_m m_t(\theta - \theta_m) - q_r(a_t + m_t)(\theta - \theta_r).$$

# The cooling process: aquaquenching

**Viscoelasticity (quasi static model)**

$$\begin{cases} -\nabla \cdot \boldsymbol{\sigma} = F \text{ in } \Omega^{\mathrm{s}} \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \\ \boldsymbol{\sigma} = \boldsymbol{K}\Big(\boldsymbol{\varepsilon}(u) - A_1(a,m,\theta)\boldsymbol{I} - \int_0^t \gamma(a,m,a_t,m_t,\theta)\boldsymbol{S}\,\mathrm{d}\tau\Big), \\ u = 0 \text{ on } \Gamma_0 \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \\ \boldsymbol{\sigma} \cdot n = 0 \text{ on } (\partial\Omega^{\mathrm{s}} \setminus \Gamma_0) \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \end{cases}$$

**Phase fractions: austenite and martensite**

$$\begin{cases} a_t = \dfrac{1}{\tau_a(\theta)}(a_{\mathrm{eq}}(\theta) - a)\mathscr{H}(\theta - A_s) \text{ in } \Omega^{\mathrm{s}} \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \\ a(T_{\mathrm{h}}) = a_{T_{\mathrm{h}}} \text{ in } \Omega^{\mathrm{s}}, \\ m_t = c_m(1-m)\mathscr{H}(-\theta_t)\mathscr{H}(M_s - \theta) \text{ in } \Omega^{\mathrm{s}} \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \\ m(T_{\mathrm{h}}) = 0 \text{ in } \Omega^{\mathrm{s}}, \end{cases}$$

# The cooling process: aquaquenching

**Temperature**

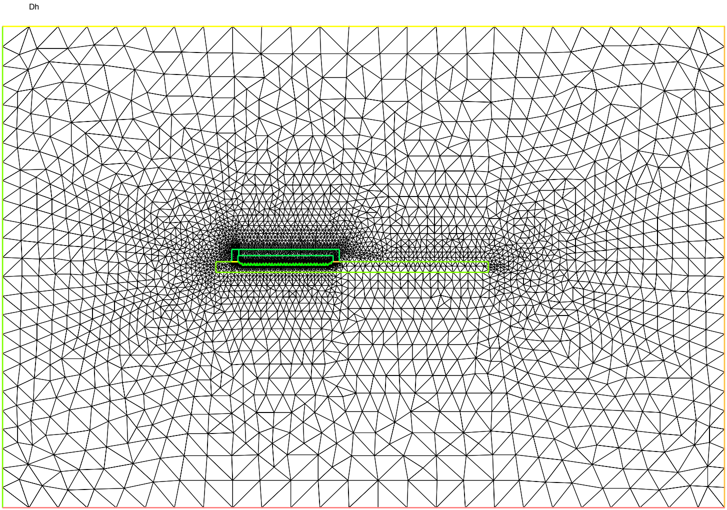
$$\begin{cases} \alpha(\theta, a, m, \boldsymbol{\sigma})\theta_t - \nabla \cdot (\kappa(\theta)\nabla\theta) \\ \qquad + 3\bar{\kappa}q(a, m)\theta\left(\nabla \cdot u_t - 3A_2(a_t, m_t, \theta)\right) \\ \quad = -\rho L_a a_t + \rho L_m m_t + A_2(a_t, m_t, \theta)\,\mathrm{tr}\,\boldsymbol{\sigma} \\ \qquad\quad + \gamma(a, m, a_t, m_t, \theta)|\boldsymbol{S}|^2 \text{ in } \Omega \times (T_\mathrm{h}, T_\mathrm{c}), \\ -\kappa(\theta)\dfrac{\partial\theta}{\theta n} = \beta(x,t)(\theta - \theta_\mathrm{e}) \text{ on } \partial\Omega \times (T_\mathrm{h}, T_\mathrm{c}), \\ \theta(T_\mathrm{h}) = \theta_{T_\mathrm{h}} \text{ in } \Omega. \end{cases}$$

$$\beta(x,t) = \begin{cases} 0 & \text{on } \partial\Omega \cap \partial\Omega^\mathrm{c}, \\ \beta_0(t) & \text{on } \partial\Omega \cap \partial\Omega^\mathrm{s}. \end{cases}$$

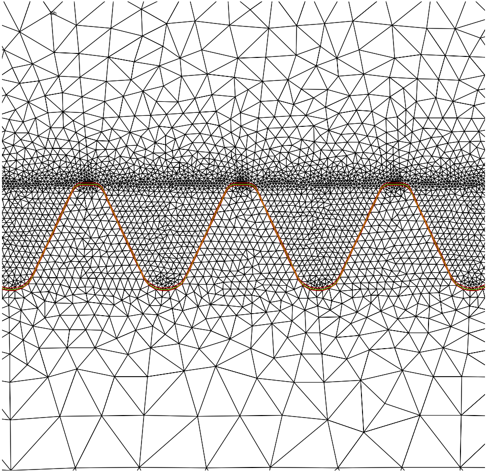
$\theta_\mathrm{e}$ is the temperature of the quenchant.

# Numerical simulation

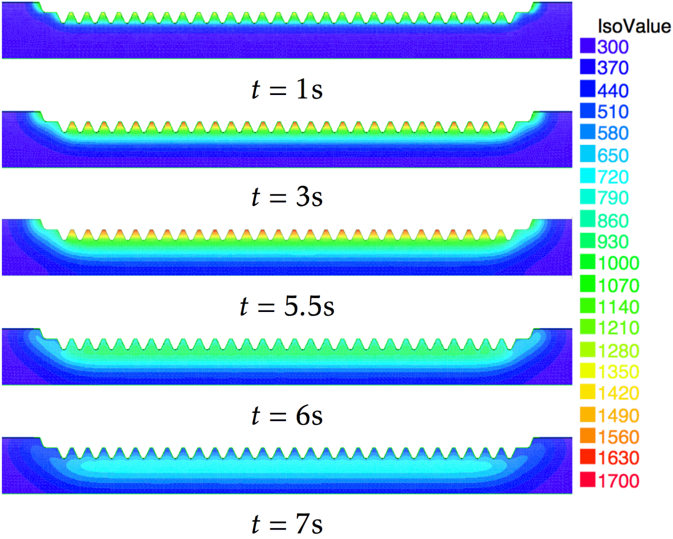
Finite elements in space (P2-Lagrange) + finite difference in time (Crank-Nicolson): 61790 elements, 30946 vertices



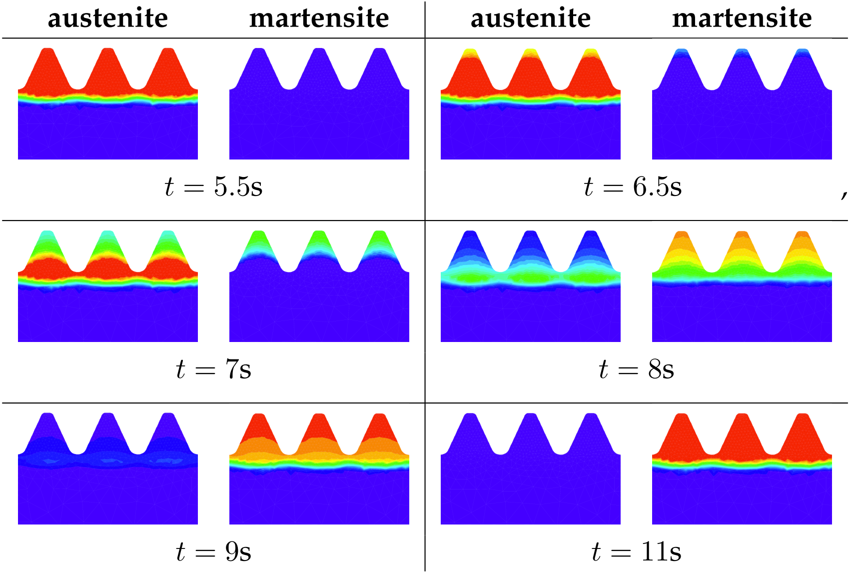
# Numerical simulation



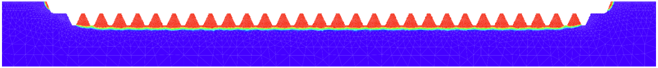
# Numerical simulation: temperature evolution



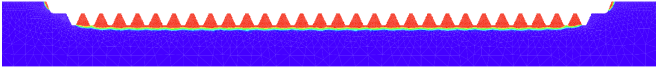
# Numerical simulation: austenite and martensite



# Numerical simulation: austenite and martensite

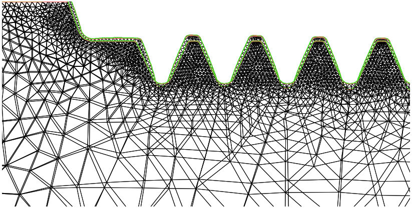


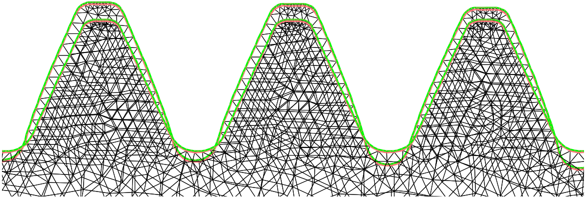
austenite at $t = 5.5$s (end of heating process)



martensite at $t = 11$s (end of cooling process)

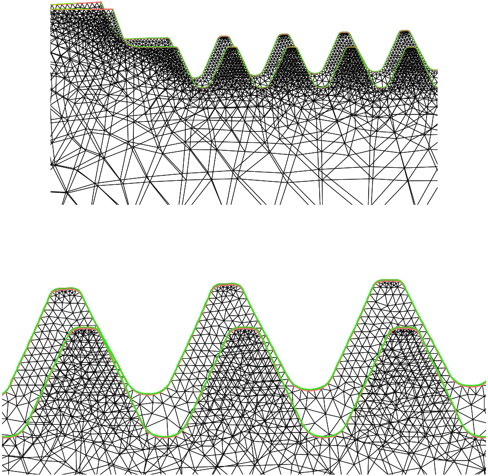
# Numerical simulation: displacements





Distorted mesh (with a scale factor of 10) after the heating stage.

# Numerical simulation: displacements



Distorted mesh (with a scale factor of 10) after the cooling stage.

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# Thank you very much for your attention!