

COLLOQUIUM Fall 2005

Edward R Rozema

Department of Mathematics

University of Tennessee at Chattanooga

Tuesday, September 29, EMCS 233, 3:10 pm.

### Modeling Epidemics

**Abstract.** Recent events have led to an increased interest in emerging infectious diseases. Mathematical modeling can contribute to our understanding on many levels by predicting the future course of the disease, clarifying relationships between classes of individuals, establishing threshold values and evaluating the effectiveness of different types of controls. This talk will apply various models to the SARS epidemic of 2003 and a measles outbreak in the Netherlands in 1999-2000. We will take an historical approach beginning with the logistic curve popularized by Raymond Pearl and Lowell J. Reed in the 1920s. We will then consider F. J. Richards' generalization in 1959 using a shape parameter. We next use systems of differential equations, an approach initiated by W. O. Kermack and A. G. McKendrick in 1927. We end with a proposed system that incorporates Richards' shape parameter.

The talk should be accessible to faculty and students who have had an introduction to differential equations.

COLLOQUIUM Fall 2005

Aniekan A. Ebiefung\*,  
U. C. Foundation Professor

Department of Mathematics

University of Tennessee at Chattanooga  
Thursday, October 6, EMCS 233, 3:15 pm.

**Leontief Input-Output Pollution System:  
An Alternative Formulation and Generalizations**

**Abstract.** An alternative formulation of the Leontiefs inputoutput pollution model is provided. It describes the flow of pollution through all sectors of the economy, and allows each sector to evaluate pollution generated by different manufacturing processes simultaneously. While the Leontiefs model requires both the technology matrix of the economy and a rectangular matrix of pollution output coefficients, the new model requires only an equivalent pollution "technology" matrix of the economy. The new model enables each sector of the economy to select and produce a product that emits the least amount of pollutants. For a given amount of pollutants emitted by a sector, a method for computing the corresponding number of items to be manufactured by that sector is provided. Examples are given to demonstrate an application of the model.

In this project, we use simple mathematic tools to solve an important economic and environmental problem. This talk should be of interest to those in economics, environmental sciences, and any body who has any concern about pollution levels in our cities.

\* Web Site: <http://www.utc.edu/Faculty/Aniekan-Ebiefung/>

Ram Verma

Department Mathematics,  
University of Toledo, Toledo, Ohio

Monday, October 17, EMCS 422, 3:30 pm.

**A-Monotone Mixed Variational Inclusion Problems  
Based on Resolvent Operator Techniques**

**Abstract.** Based on the notion of  $A$ -monotonicity [R. U. Verma, JOTA 129 (3)(2006) (in Press)] and related resolvent operator techniques, approximation solvability of a new class of nonlinear variational inclusion problems is presented. Resolvent operator techniques have been applied to nonlinear variational inclusion problems, especially to sensitivity analysis, model equilibria problems in economics, optimization and control theory, operations research, transportation network modeling, and mathematical programming. Since  $A$ -monotonicity generalizes the well-known notion of maximal monotone mappings, the results thus obtained, are general in nature.

Our next speaker will be David Ashe, Department of Mathematics, UTC, October 27.

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Charles Johnson

Department of Mathematics

College of William and Mary,  
Williamsburg, Virginia

Friday, October 21, EMCS 422, 2:00 pm.

M-matrices and (mostly) Their Inverses

**Abstract.** An  $M$ -matrix is a square matrix with nonpositive off-diagonal entries and an entry-wise nonnegative inverse. An inverse M-matrix is then any nonnegative matrix that arises as the inverse of an  $M$ -matrix. These classes arise frequently in applications and have played an important role in matrix theory. We discuss a few classical results and some recent developments about such matrices, mostly about inverse  $M$ -matrices.

The talk should be accessible to students who have taken Linear Algebra.

This talk is supported by the Center of Excellence in Computer Applications.

David J. Ashe

Department of Mathematics

University of Tennessee at Chattanooga,  
 David Leach, University of West Georgia, and  
 C. A. Rodger, Auburn University

2-Regular Leaves of Partial 8-cycle Systems

Thursday, October 27, EMCS 238, 3:15 pm.

**Abstract.** In this paper, we find necessary and sufficient conditions for the existence of a 8-cycle system of  $K_n - E(R)$  where  $R$  is any 2-regular not necessarily spanning subgraph  $R$  of  $K_n$ .

The actual theorem is stated below.

**Theorem**

Let  $R$  be a 2-regular subgraph in  $K_n$ . There exists a 8-cycle system of  $K_n - E(R)$  if and only if  $n$  is odd,  $n \neq 5, 7$ , and  $|E(K_n - E(R))|$  is divisible by 8.

Much of this theorem is proven using Lemma 1 shown below. Since Lemma 1 by itself does not completely solve the problem, other tools must be used namely Lemma 2. Lemma 1 along with Lemma 2 solve all cases for  $n \geq 25$ . The proofs of Lemma 1 and Lemma 2 are inductive in nature and therefore the small cases,  $1 \leq n \leq 23$ , must be solved by hand. This is a very generalized overview of the proof. The colloquium will go into much more detail.

**Lemma 1**

Let  $n \geq 25$  be odd. Suppose  $R$  is an  $n$ -admissible 2-regular graph which contains cycles of lengths  $\ell_1, \dots, \ell_\alpha$ , named so that for some  $x \geq 0$

$$\sum_{i=1}^{x-1} \ell_i \leq 16 \text{ and } \sum_{i=1}^x \ell_i \geq 19.$$

Suppose also that if  $R'$  is an  $(n - 16)$ -admissible 2-regular graph then there exists an 8-cycle system of  $K_{n-16} - E(R')$ .

Then there exists an 8-cycle system of  $K_n - E(R)$ .

**Lemma 2**

Suppose that  $R$  is  $n$ -admissible,  $n$  odd, and 8 divides  $|E(K_n) - E(R)|$ , and that  $R$  contains cycles  $c_1, \dots, c_\alpha$  of lengths  $\ell_1, \dots, \ell_\alpha$  respectively. Suppose further that  $\ell_i = a_i + b_i$  for  $1 \leq i \leq \alpha$  and  $x \in \{1, 2, 3, 4\}$  such that

- for  $1 \leq i \leq x$ ,  $a_i \geq 3$  and  $b_i = 0$  or  $b_i \geq 3$ ;
1. for  $x + 1 \leq i \leq \alpha$ ,  $a_i = 0$  and  $b_i = \ell_i \geq 3$ ,
  2.  $\sum_{i=1}^x a_i = 12$ , and
  3.  $n \geq 13 + 4x$ .

Suppose also that

4. if  $R'$  is an  $(n - 8)$ -admissible 2-regular graph then there exists an 8-cycle system of  $K_{n-8} - E(R')$ .

Then there exists an 8-cycle system of  $K_n - E(R)$ .

Since the proofs of Lemma 1 and Lemma 2 use induction, students that have taken or currently taking Math 300 or Math 303 might find interest in this talk.

Dušan Repovš

Institute for Mathematics, Physics and Mechanics,  
University of Ljubljana, Ljubljana, Slovenia 1001

Monday, November 7, EMCS 422, 3:15 pm.

Topology of Wild Cantor Sets in  $R^3$

**Abstract.** The first part of the talk will be a historical survey on wild Cantor sets in  $R^3$ , the first such set being constructed by Louis Antoine already in the 1920's in his Dissertation, after he was blinded while serving in the French army during WWI. In the second part of the talk we shall present a new general technique for constructing wild Cantor sets in  $R^3$  which are nevertheless Lipschitz homogeneously embedded into  $R^3$ . Applying the well-known Kauffman version of the Jones polynomial we shall show that our construction produces even uncountably many topologically inequivalent wild Cantor sets in  $R^3$ . These Cantor sets have the same number of components in the interior of each stage of the defining sequence and are Lipschitz homogenous. We shall also announce some other new results on wild Cantor sets in  $R^3$  and state some related open problems and conjectures.

Haiyan Wang\*

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Friday, November 18, EMCS 422, 2:00 pm.

**Positive Solutions of Nonlinear Systems of Differential Equations**

**Abstract.** In this talk I will start with a brief discussion of positive solutions of algebraic equations, and then introduce new concepts of superlinearity and sublinearity for nonlinear system of differential equations. I will illustrate that the number of positive solutions of boundary value problems for nonlinear systems of differential equations can be determined by appropriate combinations of superlinearity and sublinearity assumptions.

This talk is accessible to a general audience.

Marc Loizeaux

Department of Mathematics,  
University of Tennessee at Chattanooga  
Chattanooga, TN  
Tuesday, January 24, EMCS 422, 3:00 pm.

**4-critical Graphs with Diameter Two**

**Abstract.** Let  $\gamma_t(G)$  denote the total domination number of the graph  $G$ .  $G$  is said to be total domination edge critical, or simply  $\gamma_t$ -critical, if  $\gamma_t(G + e) < \gamma_t(G)$  for each edge  $e$  that is not in the edge set of  $G$ . For 4-critical graphs  $G$ , the diameter of  $G$  is either 2, 3, or 4. In a previous paper we characterized structurally the 4-critical graphs with diameter four. In this paper we study the 4-critical graphs with diameter two.

This talk is accessible to all students interested in Mathematics.

John R. Graef and Lingju Kong\*

Department of Mathematics,  
University of Tennessee at Chattanooga  
Chattanooga, TN

Tuesday, February 14, EMCS 422, 3:00 pm.

**A Necessary and Sufficient Condition for Existence of Positive Solutions  
of Nonlinear Boundary Value Problems**

**Abstract.** We study the nonlinear boundary value problem  $(\phi(u''))'' = f(t, u, u', u'')$ ,  $t \in (0, 1)$ ,  $u^{(2i)}(0) = u^{(2i)}(1) = 0$ ,  $i = 0, 1$ , and obtain a necessary and sufficient condition for the existence of symmetric positive solutions. We also discuss the application of our result to the special case where  $f$  is a power function of  $u$  and its derivatives. Our analysis mainly relies on the lower and upper solution method.

\* the speaker

The talk is accessible to students who have had an introduction to differential equations.

COLLOQUIUM Spring 2006

Sharon Brueggeman \*

Department of Mathematics,  
University of Tennessee at Chattanooga  
Chattanooga, TN

and Darrin Doud

Dept of Mathematics,  
Brigham Young University

Thursday, March 9, EMCS 422, 3:00 pm.

**Local discriminant bounds and ramification of extensions of quadratic fields**

**Abstract.** There are two main ideas in the effort to find an example of a number field with a nonsolvable Galois group, which is ramified at only one prime, and where that prime is smaller than 11. First, a polynomial of degree 5 or higher does not have to be solvable by radicals and so often describes a number field with nonsolvable Galois group. Second, the ramification of a number field can be determined from its discriminant.

There has been much work done on searching polynomials of degree  $n$  to locate a number field with the desired properties. So far, degrees 5 to 8 have been completed with no examples found. Due to the size of other search spaces, it is not feasible to go further in this direction at this time.

Instead my coauthor and I decided to investigate degree  $n$  extensions of quadratic fields. In this talk, I will discuss the techniques of discriminant bounding and prove many nonexistence results.

\* the speaker

Ronald Smith

Department of Mathematics,  
University of Tennessee at Chattanooga  
Chattanooga, TN

Tuesday, March 21, EMCS 422, 3:00 pm.

Positive, Path Product, and Inverse  $M$ -matrices

**Abstract.** Certain facts are noted, e.g., common characteristics of positive definite matrices and  $M$ -matrices as well as connections between nonnegative matrices and  $M$ -matrices / inverse  $M$ -matrices. It is known that an inverse  $M$ -matrix is strict path product. It is shown that any square positive matrix can be made strict path product via predictable additions to the diagonal and that any (normalized) strict path product matrix can be made inverse  $M$  by additions to the diagonal that are bounded in terms of the order of the matrix. The latter has implications on pairs of inverse  $M$ -matrices whose Hadamard product is inverse  $M$ . A determinantal inequality relating principal minors and certain associated almost principal minors is derived for normalized inverse  $M$ -matrices.

Most of the talk should be accessible to students who have completed Math 212.

COLLOQUIUM Spring 2006

Leigh J. Halliwell, LLC

www.lhalliwell.com,  
Chattanooga, TN

Thursday, April 6, EMCS 422???, 3:00 pm.

**Linear Regression and Actuarial Science**

**Abstract.** Common actuarial methods are deterministic versions of regression models. Mr. Halliwell will set out the matrix formulation of the regression model and briefly discuss the Gauss-Markov theorem. He will then take a simple actuarial example and discuss two common problems of model design, multicollinear and stochastic regressors. This will involve analysis of variance, orthogonal regression, and regression toward the mean.

The talk should be accessible to students who have completed Math 307.

Boris P Belinskiy

Department of Mathematics,  
University of Tennessee at Chattanooga  
Chattanooga, TN

Tuesday, April 11, EMCS 422, 3:05 pm.

**Optimal Design of an Optical Length  
of an Elastic Rod with the Given Mass**

**Abstract.** The optical length of an elastic rod appears to be important in the problems of control of its longitudinal vibrations. We consider the problem of optimal design (optimal density distribution  $\rho(x)$ ) of an elastic rod of a given variable modulus of elasticity  $k(x)$  with the optical length as a criterion assuming that the total mass  $M$  of the rod is given. The results provide us with some bounds on the optical length.

Control theory for an elastic rod governed by the wave equation seems to be complete at the moment. The conditions of exact controllability for a non-homogeneous rod were described by H. O. Fattorini and D. L. Russell. In particular, the minimal time of control appears to be equal to the double *optical length* of the rod,  $T^*$ , which is defined as the time of wave propagation along the rod and, for time independent parameters of the rod, is given by  $T^* = \int_0^l \sqrt{\rho(x)/k(x)} dx$ . We consider the problem of the optimal design of a rod of a given length  $l$  and total mass  $M = \int_0^l \rho(x) dx$ . We assume modulus of elasticity to be a given function of either coordinate or time. We consider all models of the rod studied in the literature and find maximum and / or minimum of  $T^*$  (if possible). It appears that extrema do not necessarily exist, in which case we construct the corresponding minimizing sequence.

The talk should be accessible to students who have some knowledge in Math 350.

Matt Matthews

Department of Mathematics

University of Tennessee at Chattanooga

Thursday, April 20, EMCS 422, 3:00 pm.

**Granular Flows through Inclined Hoppers:  
Experiment and Simulation Compared**

**Abstract.** Granular flow problems are challenging experimentally, with numerous observation and measurement challenges, and mathematically, with models which are ad hoc and often not well understood. For these reasons, among others, clear comparisons between a real experiment and a numerical simulation are important. In this talk such a comparison will be presented. Measurements of circulation in granular flow through a tilted right conical hopper are compared with simulations of the same experiments. By varying the angle of tilt and hopper wall friction, we may gain insight into how well existing models approximate real flows.

Joint work with Bob Behringer (Duke), John Wambaugh (Duke), and Pierre Gremaud (NCSU).

Students who have completed a differential equations course, with linear algebra, will find much of the mathematics accessible. Students who have completed a numerical analysis course will see many familiar concepts.

COLLOQUIUM Spring 2006

Miroslav Bartusek

Department of Mathematics

Masaryk University

Brno, Czech Republic

Thursday, May 11, EMCS 422, 3:20 pm.

**On noncontinuable solutions of second order differential equations**

**Abstract.** This lecture will deal with noncontinuable solutions, i.e., solutions that are defined on finite intervals only and that cannot be continued for large values of the independent argument. Sufficient conditions for the existence and for the nonexistence of such solutions are given for second order differential equations of the type

$$(g(a(t)y'))' + b(t)h(y') + r(t)f(y) = e(t).$$

<sup>1</sup> Johannes H. Hattingh <sup>\*</sup>, <sup>2</sup> Elizabeth Jonck,  
<sup>2</sup> Ernst J. Joubert and <sup>1</sup> Andrew R. Plummer

<sup>1</sup> Department of Math and Statistics, Georgia St. Univ., USA,

<sup>2</sup> Department of Math, Univ. of Johannesburg, South Africa

Thursday, June 1, EMCS 422, 3:20 pm.

**Nordhaus-Gaddum Results for Restrained Domination  
and Total Restrained Domination in Graphs**

**Abstract.** Let  $G = (V, E)$  be a graph. A set  $S \subseteq V$  is a total restrained dominating set if every vertex is adjacent to a vertex in  $S$  and every vertex of  $V - S$  is adjacent to a vertex in  $V - S$ . A set  $S \subseteq V$  is a restrained dominating set if every vertex in  $V - S$  is adjacent to a vertex in  $S$  and to a vertex in  $V - S$ . The total restrained domination number of  $G$  (restrained domination number of  $G$ , respectively), denoted by  $\gamma_{tr}(G)$  ( $\gamma_r(G)$ , respectively), is the smallest cardinality of a total restrained dominating set (restrained dominating set, respectively) of  $G$ . We bound the sum of the total restrained domination numbers of a graph and its complement, and provide characterizations of the extremal graphs achieving these bounds. It is known that if  $G$  is a graph of order  $n \geq 2$  such that both  $G$  and  $\overline{G}$  are not isomorphic to  $P_3$ , then  $4 \leq \gamma_r(G) + \gamma_r(\overline{G}) \leq n + 2$ . We also provide characterizations of the extremal graphs  $G$  of order  $n$  achieving these bounds.

\* Speaker