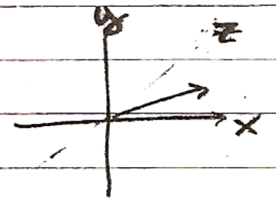
Velocity Distribution and Speed

Probability of velocity in volume element $dv_x dv_y dv_z$ at $v_x v_y v_z$

Maxwell Boltzman distribution of velocities in three dimensions

$$F(v_x, v_y, v_z)\, dv_x dv_y dv_z = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp(-mv^2/2kT)\, dv_x dv_y dv_z$$



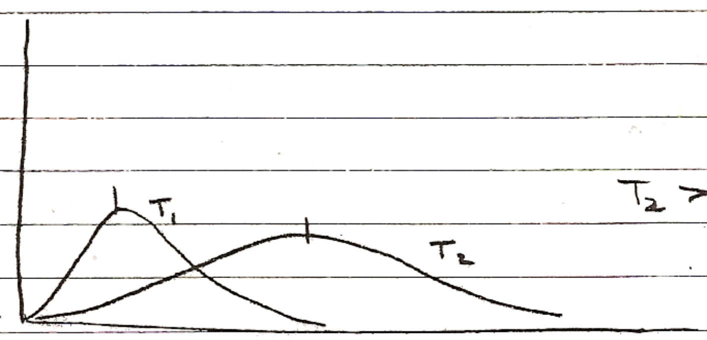
$v^2 = v_x^2 + v_y^2 + v_z^2$

if only interested in speeds then look at sphere of thickness dv

volume $4\pi v^2 dv$

$$F(v)dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp(-mv^2/2kT)\, dv$$

Maxwell distribution of speeds



- Types of molecular speeds

Give if need

root mean square speed $c = \sqrt{\langle v^2 \rangle}$ $\quad c = (3kT/m)^{1/2}$

$\left(\int_0^\infty v^2 F(v)\, dv\right)^{1/2}$

average (mean) speed $\langle v \rangle = \int_0^\infty v F(v)\, dv = \quad \bar{c} = \left(\frac{8kT}{\pi m}\right)^{1/2}$

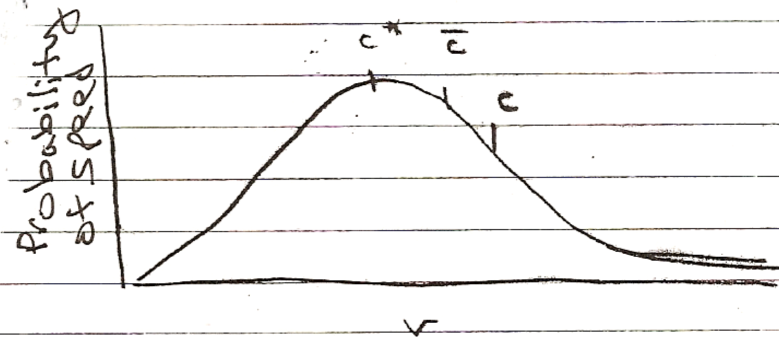
most probable speed maximum in Max dist $\quad c^* = \left(\frac{2kT}{m}\right)^{1/2}$

Molecular Speed

$c / \bar{c} / c^*$ — $1.00 / .92 / .82$

c root mean square
$\bar{c}$ ~~average~~ (mean) speed
c^* most probable speed

	He	NH_3	O_2	Hg
$\bar{c}$ (m/s)	1256	609	444	177



- **Setup to calculate $\bar{c}$ mean speed:**

$$\bar{c} = \int_0^\infty v\, F(v)\, dv$$

$$4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3\, e^{-\frac{mv^2}{2kT}}\, dv$$

-skip-

$$4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2}\left(\frac{2kT}{m}\right)^2$$

$$\bar{c} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

note that

from KMT derivation get

root mean square speed:

$$pV = \frac{nLm\langle v^2\rangle}{3} = \frac{nLm\, c^2}{3} = \frac{nLm}{3}\left(\frac{3kT}{m}\right)$$

$$pV = nLkT$$

$$\boxed{pV = nRT}$$