Numbers and Calculations

Topics include: Metric System, Exponential Notation, Significant Figures, and Factor-Unit Calculations

Refer to Chemistry 123 Lab Manual for more information and practice.
Learn metric prefixes and base units.

Numbers and Metric System

Science has to assign numbers to physical properties to test theories and repeat experiments

Science and all countries except US use International System of Units (SI units)

7 Base Units

Base units:	**Unit Name**	**Unit Symbol**
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Electric Current	Ampere	A
Luminous Intensity	Candela	cd

Any measurement can be expressed in these units

Derived Units			
Velocity	m/s	$m\ s^{-1}$	
Acceleration	m/s^2	$m\ s^{-2}$	
Force	kgm/s^2	$kg\ m\ s^{-2}$	N (Newton)
Energy	kgm^2/s^2	$kg\ m^2\ s^{-2}$	J (Joule)
Volume	m^3		

Use other modified but commonly used (older) units if needed

Common (older) Units		
Volume	Liter	$L = dm^3$
Length	Angstrom	$A = .1nm = 10^{-10}m$
Pressure	Atmosphere	atm
Energy	Calorie	cal
Volume	Milliliter	mL

Volume	Cubic centimeter	cm^3 = mL

Use conversion factors to convert from English to metric

2.54 cm = 1 inch
m = 39.4 in
454 g = 1 lb (pound)
kg = 2.2 lb
1 L = 1.06 qt (quart)

Prefixes

Metric System prefixes are based on powers of ten
In order to span the range from large (planets) to small (atoms)

Prefix	Abbreviation	Factor	Scientific Notation
Giga	G	1,000,000,000	10^9
mega	M	1,000,000	10^6
kilo	k	1,000	10^3
hecto	h	100	10^2
deka	da	10	10^1
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.0000000000001	10^{-12}

Examples of length:
1 Mm = Chattanooga-Tampa 10^6 m ~ 620mi
1 km = .62 miles
1 m = meter stick
1 mm = width of paper clip
1 μm = size of particles that are harmful to lungs (cigarette smoke)
1 nm = size of small molecule

Exponential Notation

Exponents and Operations
1000 = $1\text{x}10^3$ .001 = $1\text{x}10^{-3}$ 2000 = 2 x 10^3

Multiply (add exponents)

$(10^3)(10^3) = 10^{3+3} = 10^6$
$(10^3)(10^{-3}) = 10^{3+-3} = 10^0 = 1$

Divide (subtract exponents)

$(10^3)/(10^3) = 10^{3-3} = 10^0 = 1$

Power (multiply exponents)

$(10^3)^3 = 10^9$
$(2 \text{ x } 10^3)(3 \text{ x } 10^6) = 6 \text{ x } 10^9$
$(2 \text{ x} 10^3)/ (3 \text{ x } 10^6) = 2/3 \text{ x } 10^{3-6} = .66 \text{ x } 10^{-3} = 6.6 \text{ x } 10^{-4}$

Addition and Subtraction (only if to the same power)

$(6.2 \text{ x } 10^3) – (0.2 \text{ x } 10^3) = 6.0 \text{ x } 10^3$

$(3000)/(15) = 3.0 \text{ x } 10^4/ 1.5 \text{ x } 10^1 = 2.0 \text{ x } 10^3$

Why use exponents?
Atoms are so small that there are huge number of atoms in even small amount of matter

Ex 18.0g of water contains $6.02 \text{ x } 10^{23}$ molecules!
And you don't want to have to write 602 000 000 000 000 000 000 000

Significant Figures (sig fig)

Number written to correctly show the precision of number

Mass of Penny	Significant Figures	Precision
3 g	1	3 + or - 1
3.1 g	2	3.1 + or - 0.1
3.12 g	3	3.12 + or - 0.01
3.120 g	4	3.120 + or - 0.001
3.1204 g	5	3.1204 + or - 0.0001

But 0.0023 has only 2 significant figures (zeros to left of first digit don't count for sig fig)

Zero to right is significant 3.120 g (4 sig figs) or 10002 (5 sig figs)
Zero to left of number is not significant 0.003 kg (1 sig fig)

310 so write as $3.1 \text{ x } 10^2$
300 so write as $3 \text{ x } 10^2$

Round off
5, 6, 7, 8, 9 round up 3.125→ 3.13
0, 1, 2, 3, 4 round down 3.124→3.12

<u>Significant Figures in Computations</u>

+ or - no sig fig to right of number with least number of decimal places

161.3
<u>+ 5.61</u>
166.91
166.9 can not go past the uncertainity in 0.3 place

x or / no more sig fig than least precise number

161.3
<u>x 5.61</u>
904.893
905 since only 3 sig fig in 5.61 then answer can only be 3 sig fig

<u>Scientific Notation</u>

Write number as exponent to indicate correct sig fig

Write not as 29,979,000,000 cm/s but instead as 2.979×10^{10} cm/s or 2.979×10^{10} cm s^{-1}

<u>Why are metric conversions important</u>

NASA Says Metric Mixup Doomed Mars Spacecraft
Summary: The $125 million spacecraft that was destroyed on a mission to Mars was probably doomed by NASA scientists' embarrassing failure to convert English units of measurement to metric ones. The Mars Climate Orbiter flew too close to Mars and is believed to have broken apart or burned up in the atmosphere. NASA's Jet Propulsion Laboratory said that its preliminary findings showed that Lockheed Martin Astronautics in Colorado submitted acceleration data in English units of pounds of force instead of the metric unit called Newtons, At JPL, the numbers were entered into a computer that assumed metric units.

Ex. How many newtons in 1 pound of force?

force is mass times acceleration
? N = 1 lb
? N = 1 lb (454g/ lb)(1kg / 1000g)(9.807 m/s^2)($N/kgms^{-2}$) = 4.45 N

Factor-Unit Method (Dimensional Analysis)

Include unit with number and use conversion factors to change units
1. Set up problem
2. Write conversion factors so units cancel
3. Do math to obtain answer
4. Write in scientific notation (check significant figures)

Ex. How many inches in 1.2 miles?

? in = 1.20 mi(1760 yd/mi)(3ft/ yd)(12 in/ ft)   numerator/denominator
= 76032 in
= 7.60×10^4 in

Ex. How many cm in 1.2 km?

? cm = 1.20 km (10^3m/ km)(10^2 cm/ m)
= 1.20×10^5 cm

Ex. How many cm in exactly 1.000 yard? 2.54 cm = 1 in

? cm = 1.000 (3 ft/ yd)(12 in/ ft)(2.54 cm/ in)   exact numbers do not limit sig fig
= 91.44 cm

Ex. 10 km/hr is how many m/s?
work on numerator and denominator (top and bottom) separately

? m/s = 10 km/hr (hr/60 min) (min/ 60s) (10^3 m/ km)
= 2.8 m/s

Density Problem

Density = Mass/ Volume

Density used as conversion factor values can be found in tables
Mg density = 1.74 g/cm^3 H_2O density = 1.0 g/cm^3

Ex. What is mass of 2.3m^3 of Mg (Magnesium)?
? kg = 2.3m^3 (10^2 cm/m)^3 (1.74 g/cm^3) (10^{-3} kg/g)
= 4.0 x 10^5 kg

More Factor Unit Problems

Ex. How many grains of sand at Daytona Beach?
assuming 1 grain has volume of 1 mm^3
and beach is
10 mi long ~ 16 km
100 ft wide ~ 30 m
30 ft deep ~ 9.1 m

Volume = 16km (10^{3}m/km) x 30m x 9.1m = 4.4 x 10^6 m^3

Grains = (4.4 x 10^6 m^3) (10^3 mm/m)^3 (grain/mm^3) = 4.4 x 10^{15} grains
m^3 → mm^3 → grains be sure to cube number and units

Below are calculations to convert English units to metric

? m = 10 mi (5280 ft/ mi)(12 in/ft)(2.54 cm/in)(m/ 100 cm) = 16093 m = 1.6 x 10^4 m

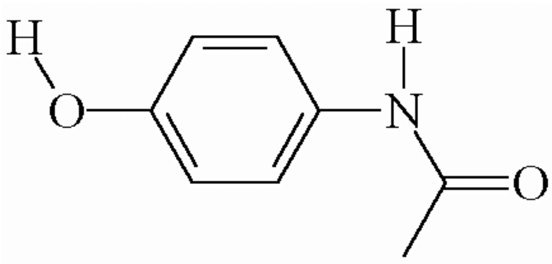
? m = 100 ft (12in/ ft)(2.54 cm/ in)(m/ 100cm) = 30.480 = 3.0 x 10^1 m

? m = 30 ft (12in/ ft)(2.54 cm/ in)(m/ 100cm) = 9.144 ~ 9.1 m

Answer above may not be exact but shows how you can approximate complicated numbers

Ex. A child swallowed a whole bottle of Tylenol (Acetaminophen)

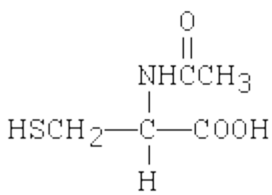
Tylenol's toxins damage the liver



acetaminophen

(http://www.chem.purdue.edu/gchelp/116exams/solut.html)

The treatment is Acetylcysteine



Acetylcysteine

(http://www.sunderland.ac.uk/~hs0dad/qm/ctinew/v2bprops.htm)

Must give the correct dosage of the antidote Acetylcysteine
(140 mg Acetylcysteine/ kg body weight)

so what would correct dose of Acetylcysteine be for 40lb child?

? g antidote = (40 lb)(454 g/ lb)(kg/ 1000 g)(140 mg antidote/ kg)(g/ 10^3 mg)
= 2. 54 g of acetylcysteine antidote

Ex. How molecules of water in a cup of water (8 oz)

? molecules = (8 oz H_2O)(1lb/ 16 oz)(454 g/1 lb)(mol/ 18 g)(6.02×10^{23}/mol)
= 7.6×10^{24}

Other big numbers are small by comparison
People on earth ~ 6×10^9 ~ 6 billion
Stars in our galaxy ~ 4×10^{11} ~ 400 billion

Final math note – Logarithms

Logarithm is number that when (10 or e) is raised to that power gives desired number.
The log of 1000 is 3 because $10^3 = 1000$.
The log of 2000 is 3.301 because $10^{3.301} = 2000$
The ln of 1000 is 6.908 because $e^{6.908} = 1000$

Common logarithm is log ex. $\log(10^3) = 3$
Natural logarithm is ln ex. $\ln(e^3) = 3$ e is special number = 2.71828

A logarithm has two parts the characteristic and the mantissa. The mantissa tells what the number is and the characteristic tells the power of ten or how big the number is.

For example the log of 1288 is log(1288) = 3.110 which means $10^{.110} \times 10^3 = 1.288 \times 10^3$
with 3 = characteristic and .110 = mantissa

Working a problem with log (when multiplying numbers you add logs)
$(56 \times 23) = 10^{1.748} \times 10^{1.362} = 10^{1.748 + 1.362} = 10^{3.110} = 1288$

Working with same problem with ln
$(56 \times 23) = e^{4.025} \times e^{3.136} = e^{4.025 + 3.136} = e^{7.161} = 1288$

Prior to use of calculators, logarithms were routinely used for calculations. This is mostly taken care of for you now, but should understand. Calculator has the functions:

ln x and e^x and these are opposite operations ex: $\ln(e^2) = 2$

log x and 10^x and these are opposite operations ex: $\log(10^2) = 2$

the ln of a number is approximately 2.3026 times the log of same number
ex: log (1000) = 3.000 but ln (1000) = 6.9078
and 6.9078 = (2.3026) (3.000)