9 Exponential Growth/Decay

Definition. $f(x) = b^x$, where $b > 0$, $b \neq 1$ is called an *exponential function with base b and exponent x*. Its domain is the set of real numbers.

Laws of exponents. Let b, c be positive numbers and x, y be real numbers.

1. $b^x b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(bc)^x = b^x c^x$
5. $\left(\dfrac{b}{c}\right)^x = \dfrac{b^x}{c^x}$

Definition. Let $b > 0$, $b \neq 1$. The *logarithm to the base b of x*, denoted by $\log_b x$, is the power to which b must be raised to produce x. Therefore

$$y = \log_b x \qquad \Leftrightarrow \qquad b^y = x.$$

Thus, we see that a logarithm is an exponent (this is the basis for converting log expressions to exponential expressions and vice versa). Also, this implies that logs are defined for positive values of x only.
$\log_{10} x$ is called the *common logarithm* of x, and is denoted by $\log x$.
$\log_e x$ is called the *natural logarithm* of x, and is denoted by $\ln x$.

Laws of Logarithms. Let b, r, s be positive numbers, with $b \neq 1$,

1. $\log_b rs = \log_b r + \log_b s$
2. $\log_b \dfrac{r}{s} = \log_b r \log_b s$
3. $\log_b r^s = s \log_b r$
4. $\log_b 1 = 0$
5. $\log_b b = 1$
6. $\log_b b^s = s$

Examples. Solve for x.

a. $\log_3 x = 4$
b. $\log 4 = x$
c. $\log_x 8 = 3$
d. $2e^{x+2} = 5$
e. $5 \ln x + 3 = 0$
f. $e^{0.4x} = 8$

g. $2e^{-0.2x} - 4 = 6$

In the physical and natural sciences it is often observed the rate of growth (rate of decay) is proportional to the number or amount y present at any time t. That is,

$$\text{Growth Equation:} \qquad \frac{dy}{dt} = ky, \quad y(0) = y_0$$
$$\text{Decay Equation:} \qquad \frac{dy}{dt} = -ky, \quad y(0) = y_0$$

where $k > 0$ and y_0 is the initial amount present. Solving, we obtain the (exponential) growth equation

$$y = y_0 e^{kt}$$

or the (exponential) decay equation

$$y = y_0 e^{-kt}.$$

Radioactive substances decay exponentially according to the decay equation $y = A(t) = A_0 e^{-kt}$ where A_0 is the initial amount present and $k > 0$ is the decay constant. The half-life of a radioactive substance is the time required for the given amount to be reduced by one-half.

The amount of Carbon-14 (C-14), a radioactive isotope of carbon, in a living organism is constant. But, when it dies it stops absorbing new quantities of C-14 and the amount of C-14 in the remains diminishes due to the natural decay of the radioactive substance. So the approximate age of a fossil can be determined by measuring the amount of C-14 it contains at present.

Example. C-14 has a half-life of 5770 years.

a. What is its decay constant?
b. If pottery from an archeological dig has one-fifth the amount of C-14 that it originally contained, approximately how old is the pottery?
c. If a skull is $15,000$ years old, what percentage of its original C-14 remains?
d. If there was 50g of C-14 present after 8000 years, how much was present initially?

Example. The half-life of radium is 1600 years. Assume there was 200 milligrams present originally in an organism.

a. Determine its decay constant.
b. When will there be 40 milligrams left?
c. How much will be left after 800 years?

Homework. Logs recovered from an archaeological dig contain 30% of the C-14 they originally contained. How long ago did the trees die?

Homework. The skeletal remains of the Pittsburgh Man were discovered in Pennsylvania. It was determined that the remains had lost 82% of the C-14 they originally contained. How old were the bones?

Homework. Phosphorus 32 has a half-life of 14.2 days.

a. If 500g of this substance was present in a material initially, what is the amount present after t

b. What is the amount present after a week?

c. How long will it be before the material has lost 70% of its Phosphorus 32?

Homework. The radioactive element polonium has a decay constant of 0.00495 where time t is measured in days.

a. Determine its half-life.

b. If there were 280g present after 20 days, how much is present initially?

c. How much will be present after 100 days?

Homework. Strontium 90 is a radioactive isotope of strontium and its decay constant is -0.0257, where the time t is measured in years.

a. Determine the half-life of Strontium 90.

b. What percentage of the original amount of Strontium 90 will be left after 10 years?

c. How long will it take for 80% of the Strontium 90 to decay?

Homework. As altitude increases, pressure decreases. Specifically, the atmospheric pressure y in millibars (mb) at a given altitude of x meters is given by $y = p(x) = Ce^{-kx}$, where $k > 0$.

a. The atmospheric pressure at 0 meters is 1013mb and at 10,000 meters is 265mb. Find the atmospheric pressure $p(x)$ at an altitude of x meters.

b. Find the atmospheric pressure at 20,000 meters.

c. If the atmospheric pressure is 55mb, what is the altitude?

10 Logistic Growth Functions

Frequently, the rate of growth is rapid at first, but decreases thereafter and approaches a constant rate. This is often modeled with the logistic growth function

$$y = Q(t) = \frac{A}{1 + Be^{-kt}}$$

where A, B, $k > 0$.

Example. The number $Q(t)$ of sailors on a destroyer who contracted a virus after t days during an epidemic is approximated by

$$Q(t) = \frac{500}{1 + 124e^{-kt}}.$$

10 sailors contacted the virus by day 4.

a. Determine the constant k.

b. How many contacted the virus in a week?

c. How long will it be before 200 of the men have caught the virus? Answer this question algebraically and graphically.

d. How many will catch the virus in the long run?

Homework. During flu epidemic the number of children in the Woodbridge Community School System who contacted influenza after t days was given by

$$Q(t) = \frac{1000}{1 + 199e^{-0.8t}}.$$

a. How many children were stricken by the flu after the 1st day?

b. How many children had the flu after 10 days?

c. How many children eventually contracted the disease?

d. How long was it before 100 children had the disease?

Homework. The U.S. population is approximated by the function

$$Q(t) = \frac{616.5}{1 + 4.02e^{-0.5t}}.$$

where $Q(t)$ is measured in millions of people and t is measured in 30 year intervals with $t = 0$ corresponding to 1930.

a. What is the expected population of the U.S. in 2020 ($t = 3$)?

b. What is the expected population of the U.S. in 2050?

c. In what year will the population of the U.S. be 500 million?

d. What will be the U.S. population eventually?

11 pH

The hydrogen ion concentration of a substance is related to its acidity and basicity and varies over a wide range. For this reason logarithms are used to create a compressed pH scale defined as follows

$$\text{pH} = \log \frac{1}{H}$$

where H is the hydrogen ion concentration in moles per liter. Pure water has a pH of 7 and is considered neutral. Substances are acidic if they have a pH less than 7 and basic if they have a pH greater than 7.

Example. Compute the pH of each substance to the nearest tenth, given its hydrogen ion concentration.

a. Seawater, 4.63×10^{-9} moles/liter;
b. Vinegar, 9.32×10^{-4} moles/liter.

Example. The most acidic rainfall ever recorded had a pH of 2.4. What was the hydrogen ion concentration to three significant digits?

Homework. Find the pH of each substance given its hydrogen ion concentration.

a. milk, 2.83×10^{-7} moles/liter;
b. garden mulch, 3.78×10^{-6} moles/liter.

Homework. What is the hydrogen ion concentration of:

a. corn, pH = 3.1;
b. grapes, pH = 4.0;
c. lemons, pH = 2.3;
d. rainwater, pH = 5.7;
e. apples, pH = 3.1;
f. oranges, pH = 3.5;
g. cherries, pH = 3.6;
h. pears, pH = 3.8;
i. tomatoes, pH = 4.2;
j. pumpkins, pH = 5.0;
k. cabbage, pH = 5.3;
l. asparagus, pH = 5.6;
m. potatoes, pH = 5.8.

12 Rate of Change

Let $y = f(x)$. The *average rate of change* of f on the interval $[a, b]$ is $\dfrac{f(b) - f(a)}{b - a}$.

Example. Let $f(x) = x^2 - 2x - 15$.

a. Find the average rate of change of f on the interval $[-4, -1]$.
b. Find the average rate of change of f on the interval $[2, 6]$.

Graphically, what does the average rate of change on $[a, b]$ represent?

Homework. Find the average rate of change of $f(x) = x^2 - x$ on

a. $[1, 3]$;
b. $[-5, -1]$;
c. $[2, 6]$.

The *(instantaneous) rate of change* with respect to x is given by the derivative

$$\frac{dy}{dx} = f'(x) = \text{the slope function.}$$

Example. Find the derivative of $y = x^2 - 2x - 15$ and use it to find the instantaneous rate of change at $x = -2$ and $x = 4$.

Homework. Find the derivative of $y = 2x^2 - x$ and use it to find the instantaneous rate of change at $x = 2$, $x = -3$, and $x = 4$.
Let $h(t)$ describe the position of an object as a function of the time t. Then,

$$\begin{aligned} v(t) = h'(t) &= \text{the velocity of the object} \\ &= \text{the rate of change of position;} \\ \\ a(t) = v'(t) = h''(t) &= \text{the acceleration of the object} \\ &= \text{the rate of change of velocity.} \end{aligned}$$

Example. Let $h(t) = -16t^2 + 64t + 128$. Find $v(t)$ and $a(t)$.

13 Free-Falling Objects

If $F'(x) = f(x)$ on some interval I, then

$$y = \int f(x)dx = F(x) + C$$

is a family of functions satisfying the differential equation (DE)

$$\frac{dy}{dx} = f(x)$$

on the interval I. If we are given an initial condition (a point on the graph), a particular solution of the DE can be obtained.

Example. $y' = 2x + 1$; $P(1, 3)$ lies on the graph. Find y.

Homework. $y' = 4x + 3$; $P(2, 5)$ lies on the graph. Find y.

Problem. An object is thrown upward from an initial height h_0 with an initial velocity of v_0 feet/sec. (We assume the positive direction is away from the Earth and that the acceleration due to gravity is $a(t) = 32$ft/sec/sec.) Then, for this free-falling body, the position equation (equation of motion) and the velocity equation can be derived and are
Equation of Motion: $y(t) = h(t) = -16t^2 + v_0 t + h_0$, $t \geqslant 0$
Velocity Equation: $y'(t) = v(t) = -32t + v_0$, $t \geqslant 0$

Key Questions:

1. What is the maximum height?
2. What is the terminal velocity (velocity upon impact)?
3. What is the average velocity over a given time interval?
4. How long does it take for the object to hit the ground?

Example. A ball is thrown upward from the top of a 160-ft high building with an initial velocity of 48 feet per second.

a. Give the velocity equation.
b. Give the position equation.
c. Find the apex.
d. Find the velocity when it strikes the ground.
e. Find the average velocity in the first 2 seconds of its downward flight.
f. Find the average velocity in the last 3 seconds of flight.
g. How long does it take for the ball to hit the ground?

Homework. A ball is thrown upward from the top of a 960-ft high building with an initial velocity of 112 feet per second.

a. Give the velocity equation.

b. Give the position equation.
c. Find the apex.
d. Find the velocity when it strikes the ground.
e. Find the average velocity in the first 2 seconds of flight.
f. Find the average velocity in the last 3 seconds of flight.
g. How long does it take for the ball to hit the ground?

Homework. A ball is thrown upward from the top of a 1280-ft high building with an initial velocity of 32 feet per second.

a. Give the velocity equation.
b. Give the position equation.
c. Find the apex.
d. Find the velocity when it strikes the ground.
e. Find the average velocity in the first 2 seconds of its downward flight.
f. Find the average velocity in the last 3 seconds of flight.
g. How long does it take for the ball to hit the ground?

14 Linear Cost/Revenue/Profit Models

Let x denote the number of units of a product manufactured or sold. The *total production cost function* $C(x)$ gives the total cost of manufacturing x units of the product.

Suppose a firm has a fixed *set-up cost* of F dollars, and a *production cost* of c dollars per unit, then the total production cost function is given by

$$C(x) = cx + F.$$

Example. Compudisk, a computer disk manufacturer, sells each disk it produces (so that there is never any inventory). Thus, if x is the number produced, x also equals the number sold. There is a (set-up) cost of \$25 and each disk costs \$5 to produce.

a. The total production cost for producing x disks is given by the model $y = C(x) = \dots$
b. Make a rough graph of this equation.
c. How does the set-up cost relate to the graph of the equation?
d. How does unit cost relate to the graph of the equation?
e. What is the production cost if 20 disks are produced?
f. How many disks were produced if the total cost is \$175?
g. If Compudisks budget limits it to a total production cost of \$500, what is the maximum number of disks it can produce?

The *revenue function* $R(x)$ is the total revenue realized from the sale of x units of the product. If the selling price per unit (*unit price*) is p, the the revenue function is given by

$$R(x) = px.$$

Example. Consider the previous example, and suppose Compudisk sells each disk for \$10 each.

h. The revenue function is ...
i. Calculate the revenue if the company produces and sells 15 disks.
j. How many disks were sold if the total revenue is \$450?

The *profit* equals the revenue minus the cost, so that the *profit function* is given by

$$P(x) = R(x) - C(x) = (p - c)x - F.$$

Example. With regard to the previous examples,

k. Determine the profit $y = P(x)$.
l. Determine the profit from 10 disks.
m. How many disks were produced and sold if the profit is \$500?

The *break-even point* is the level of production where the revenue equals the total production cost, that is, where the profit is equal to zero. To find the break-even point, we must solve the equation $P(x) = 0$.

Example. With regard to the previos examples,

n. Graph the cost and revenue equations on the same set of axes.

o. What does the break-even point mean geometrically (graphically) in terms of the cost and revenue equations?

p. Determine the break-even point algebraically.

q. Give the intervals for x when there is a (positive) profit and the intervals where there is a loss (negative profit).

Homework. A company producing computers has total production costs given by $y = C(x) = 50x + 40,000$ dollars and total revenue given by $y = R(x) = 850x$ dollars where x is the number produced and sold.

a. What does 50 represent?

b. What does $40,000$ represent?

c. What does 850 represent?

d. Give the profit function.

e. Use the profit function to determine the production level at which the company breaks even.

f. Give a table of values for the profit for production levels of 10, 20, 30, 40, 50.

Homework. A manufacturer has a monthly fixed cost of \$40,000 and a production cost of \$8 for each unit produced. The product sells for \$12/unit.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

d. Compute the profit (loss) corresponding to production levels of 8000 and $12,000$ units.

e. Determine the break-even point of production.

f. Determine the intervals where the production level yields a profit or a loss.

Homework. Auto-Time, a manufacturer of 24-hour variable timers, has a monthly fixed cost of \$48,000 and a production cost of \$8 for each timer manufactured. The timers sell for \$14 each.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

d. Compute the profit (loss) corresponding to production levels of 4000, 6000, and $10,000$ timers, respectively.

e. Determine the break-even level of production.

f. Graph the cost and revenue functions and give the intervals where the production level yields a profit or a loss.

Definition. A function that is defined by more than one rule is called a *piece-wise function.* An example of piecewise function is

$$f(x) = \begin{cases} 2x - 1 & 0 \leqslant x \leqslant 3 \\ x + 2 & 3 < x \leqslant 7. \end{cases}$$

In Cost/Revenue/Profit Models, piece-wise functions are used when the cost per unit (or the selling price per unit) varies depending on the amount x of units produced/sold.

Example. You decide to buy a salad for lunch. The cost is \$3.00 for the first 4 ounces or less, plus 50 cents an ounce for anything above 4 ounces.

a. Give the function $C(x)$ representing the cost of x ounces of salad where $x > 0$.

b. Graph $C(x)$ by hand.

Example. You park downtown where the daily rates are \$2 for the first hour or fraction thereof, plus \$1.20 per hour for anything above 1 hour up to a maximum daily cost of \$8. (you have to be out after 24 hours.)

a. Give the function $C(x)$ which represents the daily cost of x hours of parking.

b. Graph $C(x)$.

Homework. A steel company charges \$5 per pound for the first 2000 pounds ordered, \$4 per pound for everything over 2000 pounds but not more than 5000 pounds and \$3 for everything over 5000 pounds.

a. Give the function $C(x)$ representing the charge for x pounds of steel ordered.

b. Graph $C(x)$.

15 Quadratic Cost/Revenue/Profit Models

Typically, as the unit price p increases, the demand x decreases, due to the fact that fewer people are willing to buy at a higher price. Thus, the unit price for a product does not remain constant and is related to the demand (the number sold).

Example. Suppose $p = 120 - 6x$ or $x = 20 - \frac{p}{6}$. Then, each $6 increase in price brings about a 1-unit decrease in demand.

Example. The research department in a small company that manufactures VCRs has established that the unit cost of each VCR is $40, the fixed cost is $1100, and the demand equation is $p = 200 - 5x$ dollars where p is the unit price and x is the number sold, $0 \leqslant x \leqslant 40$.

a. Determine the revenue function, the cost function, and the profit function.
b. Using your calculator, graph the cost and revenue functions on the same set of axes.
c. Use the cost and revenue functions to determine the break-even points algebraically and check your answer on your calculator.
d. Using parts (b) and (c), determine the production levels where you sustain a loss and where you make a profit.
e. Determine the output that produces the maximum revenue, find the maximum revenue, and find the optimal price in regard to revenue (the price at which you attain maximum revenue).
f. Graph the profit function on the same set of axes. What do the break-even production levels represent in terms of this graph?
g. Determine the output that produces maximum profit, find the maximum profit, and find the optimal price in regard to profit (the price at which you obtain maximum profit).
h. Determine the average rate of change of revenue with respect to the production level x, and the average rate of change of profit with respect to the production level x as the production is increased from 5 units to 10 units.

Homework. A company produces large screen TVs and it has been determined that the unit cost is $500, the fixed cost is $14,000, and the demand equation is $p = 1400 - 10x$ where p is in dollars and x is the demand, $0 \leqslant x \leqslant 140$.

a. Determine the revenue function, the cost function, and the profit function.
b. Using your calculator, graph the cost and revenue functions on the same set of axes.
c. Use the cost and revenue functions to determine the break-even points algebraically and check your answer on your calculator.
d. Using parts (b) and (c), determine the production levels where you sustain a loss and where you make a profit.
e. Determine the output that produces the maximum revenue, find the maximum revenue, and find the optimal price in regard to revenue (the price at which you attain maximum revenue).
f. Graph the profit function on the same set of axes. What do the break-even production levels represent in terms of this graph?

g. Determine the output that produces maximum profit, find the maximum profit, and find the optimal price in regard to profit (the price at which you obtain maximum profit).

h. Determine the average rate of change of revenue with respect to the production level x, and the average rate of change of profit with respect to the production level x as the production is increased from 10 units to 20 units.

Homework. A company produces mountain bicycles which have a fixed cost of \$36,000 and a unit cost of \$600. The demand equation is $p = 2400 - 20x$ where p is in dollars and x is the number sold, $0 \leqslant x \leqslant 120$.

a. Determine the revenue function, the cost function, and the profit function.

b. Using your calculator, graph the cost and revenue functions on the same set of axes.

c. Use the cost and revenue functions to determine the break-even points algebraically and check your answer on your calculator.

d. Using parts (b) and (c), determine the production levels where you sustain a loss and where you make a profit.

e. Determine the output that produces the maximum revenue, find the maximum revenue, and find the optimal price in regard to revenue (the price at which you attain maximum revenue).

f. Graph the profit function on the same set of axes. What do the break-even production levels represent in terms of this graph?

g. Determine the output that produces maximum profit, find the maximum profit, and find the optimal price in regard to profit (the price at which you obtain maximum profit).

h. Determine the average rate of change of revenue with respect to the production level x, and the average rate of change of profit with respect to the production level x, as the production is increased from 20 units to 30 units.

16 Basic Calculator Notes

MODE menu

NORMAL SCI ENG	Numeric notation.
FLOAT 0123456789	Number of decimal places.
RADIAN DEGREE	Unit of angle measure.
FUNC PAR POL SEQ	Type of graphing.
CONNECTED DOT	Whether to connect graph points.
SEQUENTIAL SIMUL	Whether to plot simultaneously.
REAL a+bi re^θi	Real, rectangular complex, or polar complex.
FULL HORIZ G-T	Full screen, two split-screen modes.

2nd [FORMAT] menu

RectGC PolarGC	Sets cursor coordinates.
CoordOn CoordOff	Sets coordinates display on or off.
GridOff GridOn	Sets grid off or on.
AxesOn AxesOff	Sets axes on or off.
LabelOff LabelOn	Sets axes label off or on.
ExprOn ExprOff	Sets expression display on or off.

WINDOW menu

Xmin	minimum x-value.
Xmax	maximum x-value.
Xscl	distance between thick marks on x-axis.
Ymin	minimum y-value.
Ymax	maximum y-value.
Yscl	distance between thick marks on y-axis.
Xres	sets pixel resolution. If Xres is 1, the function is evaluated at every pixel on the x-axis.

To select (highlight) a setting, press ▼ ▲ ► ◄ to move the cursor to the setting you want.

ZOOM menu

1:ZBox	Draws a box to define the viewing window. 1. Move the zoom cursor to any spot you want to define as a corner of the box, 2. ENTER 3. Use ◄ ► ▲ ▼ to define the box. 4. ENTER
2:ZoomIn	Magnifies the graph around the cursor. 1. Move the zoom cursor to the point that is to be the center of the new viewing window. 2. ENTER
3:ZoomOut	Views more of a graph around the cursor. Same procedure as in **2:ZoomIn**.
6:ZStandard	Sets **Xmin**=−10, **Xmax**=10, **Ymin**=−10, **Ymax**=10.
9:ZoomStat	Redefines the viewing windows so that all statistical data points are displayed.

Entering functions

To enter a function, $y_1 = x^2+x$.	Y= X,T,Θ,n x^2 + X,T,Θ,n
To enter a function, $y_2 = 2x-2$.	Y= ▼ 2 X,T,Θ,n − 2
To select or deselect a function.	1. Y= 2. Use ▼ ▲ to move the cursor to the function you want to select/deselect. 3. ◄ ENTER

Graphing a function

To graph the selected functions.	GRAPH
To move the cursor on the Cartesian plane.	◄ ► ▲ ►
To move the cursor along a line.	TRACE ◄ ►
To move the cursor to any valid x-value on the graph in **TRACE** mode.	Enter the value, press ENTER.

To move the cursor from one function to another in **TRACE** mode.	Press ▲ ▼ .

Calculating a function

To calculate the y-value for a given x.	1. 2nd [CALC] 2. **1:value** 3. Enter a real value for x between **Xmin** and **Xmax**. 4. ENTER . The correspondent y-coordinate on the first selected function is displayed. 5. In case, use ▲ ▼ to move the cursor from function to function.
Table of Values for a function.	1. 2nd [TBLSET] 2. Enter the starting value for x, e.g. $x = -2$. 3. Enter Δx, e.g. $\Delta x = 2$. 4. 2nd [TABLE]
To find the x-intercepts of a function.	1. 2nd [CALC] 2. **2:zero** 3. **Left Bound?** is displayed in the bottom-left corner. Use ▲ ▼ to move the cursor to the desired function. 4. Use ◄ ► (or enter a value) to select a point to the left of the x-intercept. Press ENTER . 5. **Right Bound?** is displayed. Use ► (or enter a value) to select a point to the right of the x-intercept. Press ENTER . 6. **Guess?** is displayed. Use ◄ (or enter a value) to select a point near the zero of the function. Press ENTER . 7. The cursor is on the solution and the coordinates are displayed.

To find a local minimum (resp. maximum) of a function.

1. [2nd] [CALC]
2. **3:minimum** (resp. **4:maximum**)
3. **Left Bound?** is displayed in the bottom-left corner. Use [▲] [▼] to move the cursor to the desired function.
4. Use [◄] [►] (or enter a value) to select a point to the left of the point of local minimum (resp. maximum). Press [ENTER].
5. **Right Bound?** is displayed. Use [►] (or enter a value) to select a point to the right of the point of local minimum (resp. maximum). Press [ENTER].
6. **Guess?** is displayed. Use [◄] (or enter a value) to select a point near the point of local minimum (resp. maximum). Press [ENTER].
7. The cursor is on the point of local minimum (resp. maximum) and the coordinates are displayed.

To find the point of intersection of two functions.

1. [2nd] [CALC]
2. **5:intersect**
3. **First Curve?** is displayed in the bottom-left corner. Use [▲] [▼] to move the cursor to the first function. Press [ENTER].
4. **Second Curve?** is displayed in the bottom-left corner. Use [▲] [▼] to move the cursor to the second function. Press [ENTER]
5. **Guess?** is displayed. Use [◄] [►] (or enter a value) to select a point near the intersection of the two functions. Press [ENTER].
6. The cursor is on the intersection and the coordinates are displayed.

Lists

To enter a list.

1. [STAT]
2. **1:Edit**
3. Use [◄] [►] to select the desired list.
4. After entering each data value, press [ENTER].

To edit a list.	1. [STAT] 2. **1:Edit** 3. Use [◄] [►] to select the list to be edited. 4. Use [▼] [▲] to select the data value to be edited. 5. Edit the data value. Then press [ENTER].
To insert/delete a data value from a list.	1. [STAT] 2. **1:Edit** 3. Use [◄] [►] to select the list to be edited. 4. Use [▼] [▲] to select the position where to delete/insert a data value. 5. Press [DEL] to delete the data value. 6. Press [2nd] [INS] to insert a new data value. Insert the data value, then press [ENTER].
To clear a list.	1. [STAT] 2. **4:ClrList** 3. [2nd] [L1] to clear L_1. 4. [2nd] [L1] [,] [2nd] [L2] to clear both L_1 and L_2.
To find the minimum (resp. maximum) of a list.	1. [MATH] [►] NUM 2. **6:min(** (resp. **7:max(**) 3. Insert the list, e.g. [2nd] [L1] for L_1. 4. [)] [ENTER]
To find mean, variance, etc. of a list.	1. [STAT] [►] CALC 2. **1:1-Var Stats** 3. Insert the list, e.g. [2nd] [L1] for L_1. 4. [ENTER]

To perform Linear, Power, Exponential Regression between two lists, say L_1 and L_2, and store the regression line in Y_1.

1. [STAT] [▶] CALC
2. **4:LinReg(a+bx)** for Linear Regression, or
3. **A:PwrReg** for Power Regression, or
4. **0:ExpReg** for Exponential Regression.
5. [2nd] [L1] [,] [2nd] [L2] [,]
6. [VARS] [▶] Y-VARS
7. **1:Function**
8. **1:Y1**
9. [ENTER]

Storing a statistical parameter

Example. Store the sample variance Sx in **S**.

1. [VARS]
2. **5:Statistics...**
3. **3:Sx**
4. [STO▶] [ALPHA] **S**

Example. Store the correlation coefficient r in **R**.

1. [VARS]
2. **5:Statistics...**
3. [▶] [▶] EQ
4. **7:r**
5. [STO▶] [ALPHA] **R**

Appendix A.

Conics: Parabola, Ellipse, Hyperbola

PARABOLA

A *parabola* is the set of points in a plane equidistant from a fixed point (called the *focus*) and a fixed line (called the *directrix*).

The *vertex* of the parabola is the point on the parabola that is closest to the directrix. The equation of a parabola whose directrix is horizontal, and whose vertex is in $(0,0)$ is given by

$$y = ax^2$$

In this case, the coordinates of the focus are $F(0, \frac{1}{4a})$.

Reflection Property

If a ray emanating from the focus intersects the parabola, it will reflect off the parabola in a line parallel to the axis of symmetry (used to design flashlights, headlights, etc). Conversely, if a ray parallel to the axis of symmetry strikes a parabola, it is reflected to the focus (used to design solar energy devices, satellite dishes, telescopes, etc.).

A paraboloid of revolution is a surface formed by rotating a parabola about its axis of symmetry. (Similarly for ellipsoids and hyperboloids).

Example. A satellite dish is shaped like a paraboloid of revolution. The signal emanating from a satellite strikes the surface of the dish and is reflected to the receiver, a single point. At what position (in relation to the bottom of the dish) must the receiver be located if the dish is 8 feet across at its opening and 3 feet deep at the center?

Homework. The reflector of a flashlight is shaped like a paraboloid of revolution. It is 4 inches in diameter and has a depth of 1 inch. How far from the vertex should the bulb be placed so that the rays of light reflect parallel to the axis of symmetry?

Homework. The U.S. Naval Research Lab designed a giant radio telescope. Its parabolic dish has a diameter of 300 feet and a depth of 44 feet.

a. Find an equation $y = ax^2$ for a cross-section of the dish.
b. If the receiver is located at the focus, how far should it be from the vertex?

Suspension Bridges

Cables that are parabolic in shape connect the towers of suspension bridges.

Example. The Golden Gate Bridge, a suspension bridge, has 746-foot tall towers that are 4200 feet apart. The cables touch the road surface at the center of the bridge.

a. Assuming the roadway of 220 feet above the base of the towers, draw a sketch of the Golden Gate Bridge.

b. Let the origin be the midpoint of the line segment connecting the base of the two towers. Find the equation of the parabolic cable.

c. Find the distance from the cable to the roadway at a distance 500 feet from the center of the bridge.

Homework. A suspension bridge has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center of the bridge, assuming that the road is level.

ELLIPSE

An *ellipse* is the collection of all points in the plane, the sum of whose distances from two fixed points, called the *foci*, is a constant.

If the foci have coordinates $(c, 0)$ and $(-c, 0)$, respectively, and the constant sum is set equal to $2a$, then the equation of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $b^2 = a^2 - c^2$.

Reflection Property

If a source of light (or sound) is placed at one focus, the waves transmitted by the source will reflect off the ellipse and concentrate at the other focus. This is the principal behind whispering galleries (rooms with elliptical ceilings).

Example. A hall 100 feet in length is to be designed as a whispering gallery. If the foci are located 25 feet from the center, how high will the ceiling be in the center?

Homework. Jim stands at one focus of a whispering gallery. He is 6 feet from the nearest wall and his friend is 100 feet away at the other focus. What is the length of this whispering gallery and how high is its elliptical ceiling at the center?

Lithotripter

A lithotripter is a machine used to break up kidney stones without surgery by producing powerful shock waves. The patients kidney stone is placed at one focus of an ellipse, and the source of the shock waves at the other. Thus, when the shock waves are emitted, the kidney stone absorbs all the energy from the shock wave and breaks up without harming the patient, reducing both risk and recovery time.

Example. The source of a shock wave is placed at one focus of an ellipsoid with major axis (length) equal 8 inches and minor axis (length) equal 5 inches. How far should the kidney stone be placed from the source?

Homework. The source of a shock wave is placed at one focus of an ellipsoid with major axis of 10 inches and minor axis at 6 inches. How far should the kidney stone be placed from the source?

HYPERBOLA

A *hyperbola* is the collection of all points in the plane the difference of whose distances from two fixed points, called the *foci*, is a constant.

If the foci have coordinates $(c, 0)$ and $(-c, 0)$, respectively, and the constant difference is set equal to $2a$, then the equation of the hyperbola is given by

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where $b^2 = c^2 - a^2$.

Reflection Property

A ray of light or sound directed at one focus of a hyperbola will be reflected through the other focus.

The *focal length* of a hyperbolic lens is the distance from a focus to the nearest vertex.

Example. A hyperbolic reflector with focal length 10 inches is designed so that light directed toward one of its foci is reflected through a point that lies on its transverse axis 40 inches from the vertex corresponding to that foci. Assuming the hyperbola is horizontal and in standard position, find its equation.

Homework. A hyperbolic reflector with focal length 18 inches is designed so that light directed toward one of its foci is reflected through a point that lies on its transverse axis 50 inches from the vertex corresponding to that foci. Assuming the hyperbola is horizontal and in standard position, find its equation.

Appendix B.

Sound Intensity

The *loudness* $L(x)$, measured in decibels (names in honor of Alexander Graham Bell), of a sound of intensity x (measured in watts per square meter) is defined as

$$L(x) = 10 \log \frac{x}{I_0}.$$

where $I_0 = 10^{-12}$ watt/m^2 is the least intense sound the human ear can detect.

Example. At the threshold of human hearing, i.e. $x = I_0$, what is the loudness in decibels?

Example. Find the decibel rating of the following sounds:

a. whisper, 5.2×10^{-10}w/m^2;
b. jackhammer, 3.2×10^{-3}w/m^2.

Example. Find the sound intensity of each sound

a. shotgun blast, 140 decibels;
b. subway train, 100 decibels.

Homework. Find the decibel rating of the following sounds:

a. normal conversation, 3.2×10^{-6}w/m^2;
b. heavy traffic, 8.5×10^{-4}w/m^2;
c. jet plane with afterburner, 8.3×10^{-2}w/m^2;
d. threshold of pain, 1.0×100w/m^2.

Homework. Find the sound intensity of each sound:

a. power lawn mower, 100 decibels;
b. amplified rock music, 110 decibels;
c. firecrackers, 120 decibels;
d. normal conversation, 50 decibels;
e. light rainfall, 20 decibels.