Math with Toys

Georgia Mathematics Conference
Georgia Council of Teachers of Mathematics
Friday, October 16, 2009, 11:45 a.m. – 12:45 p.m.

Session Description

Participate in standards-based, mathematics activities with "toys," including paper airplanes, cars, dowel caps, seesaws, tops, cards, dice, etc., with an emphasis on active learning through data collection.

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Web page: http://oneweb.utc.edu/~deborah-mcallister/
Session handout: http://oneweb.utc.edu/~deborah-mcallister/gctm09.pdf

Toys and Activities

Standards Reference

Georgia Department of Education. (2008). *Mathematics*. Retrieved October 8, 2009, from https://www.georgiastandards.org/Standards/Pages/BrowseStandards/MathStandards.aspx
Process skills occur throughout the activities.

Useful Web Sites

NCES Kids Zone - http://nces.ed.gov/nceskids/
NCES - Create A Graph - http://nces.ed.gov/nceskids/createagraph/
Online Graph Paper - http://www.incompetech.com/beta/plainGraphPaper/
NCTM Illuminations - http://illuminations.nctm.org/
Project Interactivate - http://www.shodor.org/interactivate/
National Library of Virtual Manipulatives - http://nlvm.usu.edu/en/nav/vlibrary.html

Salute

One general and two privates are needed for this game. Each private has half of a deck of cards, using 1 (ace) through 10. When the general says, "salute," each private deals one card away from the deck and holds it face-up on his or her forehead. The general computes and states the product of the two numbers (positive only). Each private must find the value of the card on his or her forehead. After all cards are played, the student who has the most cards wins the game.

Variations

- Use integers (black is positive, red is negative).
- The jack, queen, and king cards can be counted as 11, 12, and 13, respectively, or removed from the deck.

Georgia Performance Standards
Grade 3 – M3N3 b, f, g; M3N4 a, c, e, f
Grade 4 – M4N3; M4N4 a, c; M4N7 d
Grade 6 – M6N1 g

Shape Island Mission

An organism from Shape Island has escaped from its cage! You were the last person to see it. It is up to you to draw the organism, in detail, and state its scientific name. Using number cubes, find the number of sides/angles of the body, the size of the body, the number of antennae, the number of feet, the number of mouths, and the number of tails. Draw and name the organism.

Use the following chart to find the name of the organism (assume the organism has two eyes):

Greek/Latin root, prefix, or suffix	Meaning	Color of Number Cube	Number(s) on Number Cube
anklo/anklos	angle	(red)	
antenna/antennae	external sense organ	(green)	
mono-	one	green, clear, yellow, white	1, 3, 5
bi-	two	green, clear, yellow, white	2, 4, 6
tri-	three	red	3
quad-/quadra	four	red	4
pent-/penta-	five	red	5
hex-/hexa-	six	red	6
cyclo-	circle	red	1, 2
macro-	large	blue	4, 5, 6
micro-	small	blue	1, 2, 3
plast	body	(use with macro-/micro-)	
pod/poda	foot	(yellow)	
stoma	mouth	(clear)	
uro	tail	(white)	
peri-	all around	(not used)	

Samples

Number Cube	Red	Clear	Yellow	Green	Blue	White
Value	1	4	3	2	6	5
Root/Prefix/Suffix	cyclo-	bi-	mono-	bi-	macro-	mono-
Meaning	circle for a body	2 mouths	1 foot	2 antennae	large body	1 tail

Name: Bi-antennae, bi-stoma, mono-uro, mono-poda, macro-cyclo-plast.

Number Cube	Red	Clear	Yellow	Green	Blue	White
Value	4	1	4	1	2	4
Root/Prefix/Suffix	quadranklo-	mono-	bi-	mono-	micro-	bi-
Meaning	4-sided body	1 mouth	2 feet	1 antenna	small body	2 tails

Name: Mono-antennae, mono-stoma, bi-uro, bi-poda, micro-quadranklo-plast.

(Modified from Annie Blanks, Ringgold Middle School.)
See Holt, Rinehart, and Winston. (n.d.). *Shape Island*. Retrieved October 8, 2009, from http://www.btcsmn.org/StockB/ShapeIsland.pdf

Data Sheet

Number Cube	Red	Clear	Yellow	Green	Blue	White
Value						
Root/Prefix/Suffix						
Meaning						

Name:

Number Cube	Red	Clear	Yellow	Green	Blue	White
Value						
Root/Prefix/Suffix						
Meaning						

Name:

Georgia Performance Standards
Grade 3 – M3G1 a, b; M3D1 a
Grade 4 – M4D1 d
Grade 6 – M6D1 a, e; M6D2 a, b
Grade 8 – M8D3 a, b

Plot 10 Points

Draw and label a coordinate plane on a piece of graph paper (half-inch or other). Place a piece of carbon paper over the graph paper, with the "carbon" side down. Place a piece of plain paper over the carbon paper. Fasten the papers with a paper clip, and place them on the floor, with the plain paper on top. Drop a dowel cap onto the paper, from the height of your nose. Repeat, for a total of 10 drops. Unfasten the papers. For each point that was plotted, write its coordinates, to the nearest unit. (Re-plot the point to the nearest unit.)

1. What is the distance between the two points that appear to be furthest from each other? (Use the distance formula or the Pythagorean formula, as needed.) For younger grades: Locate two points on the same horizontal or vertical line. Find the distance between the two points.
2. Connect as many points, as possible, to form a polygon. Identify acute, right, and obtuse angles.
3. Estimate the area of the polygon.
4. Estimate the perimeter of the polygon.
5. Determine the slope of one side of the polygon.

Variations

- ❖ Use Quadrant I only for younger grades.

Georgia Performance Standards
Grade 3 – M3M2 c; M3M3 a, b, c; M3M4 a, b, c; M3G1 c
Grade 4 – M4G1 b; M4G3 a, b, c
Grade 5 – M5M1 a, f
Grade 6 – M6M2 a, b, c
Grade 7 – M7N1 a, c; M7A3 a
Grade 8 – M8N1 a, c, e, f, g; M8G1 a; M8G2 a, b; M8A1 a, b, c; M8A4 a

Jumping Poppers

Turn the popper inside-out. It will jump... but when, and how high? Let's collect some data:

	Time until Jump (s)			Height of Jump (cm)		
	Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3
"Rim" at bottom						
"Belly" at bottom						

Variations

- ❖ What other conditions could be varied?
- ❖ Draw a bar graph to represent one set of data.
- ❖ Calculate the mean, median, and mode of the class data.
- ❖ Convert from centimeters to meters, or centimeters to millimeters.
- ❖ Convert from metric to U.S. Customary units (e.g., centimeters to inches).

Georgia Performance Standards
Grade 3 – M3N5 a, b, c, d, e, f, g, h; M3M2 b, c; M3D1 a, b
Grade 4 – M4N2 a, b, c, d, e; M4N4 a, b; M4N7 a; M4D1 a, c, e
Grade 5 – M5N4 a; M5M3 b; M5D1 c; M5D2
Grade 6 – M6M1; M6M2 a, b; M6D1 a, c, d, e
Grade 7 – M7D1 c, f, g

Shrinky Dinks

Materials: No. 6 plastic, scissors, paper, pencil, sand paper, colored pencils, hole punch, thread, toaster oven or oven with baking sheet, spatula

Procedure:
- Preheat the toaster oven and baking sheet to 350 degrees Fahrenheit.
- Cut a flat surface from no. 6 plastic.
- Round edges, as desired.
- Trace the original shape on a piece of paper.
- Sand one side of the shape.
- Using colored pencils, draw a design on the sanded side of the shape.
- Use the hold punch to leave a hole for attaching the shape to another object.
- Bake the shape for approximately 30 seconds.
- Use the spatula to place and remove the shape, as well as to encourage the shape to flatten.
- On paper, trace the new shape inside the original shape.
- Calculate the ratio or percentage of the new to original length, width, and area.
- Use the thread as an ornament hanger.

Related Web sites:
The magical world of shrinky dinks – http://www.shrinkydinks.com/
DLTK's Printable Shrinky Dink Patterns – http://www.dltk-kids.com/type/shrinky.htm

Georgia Performance Standards
Grade 3 – M3N3 a, c, g; M3N4 a, b, f; M3N5 c; M3M2 b, c; M3M3 a, b, c; M3M4 a, b, c
Grade 4 – M4N2 a, b, d, e; M3N4; M4N4 a, b, c; M4N5 a; M4N7 a
Grade 5 – M5N1 c; M5N2 a; M5N3 a, d; M5N4 a; M5N5 a; M5M1 a, f; M5A1 a, b
Grade 6 – M6M2 a, b, c; M6G1 c, d, e; M6A1; M6A2 b, c, d
Grade 7 – M7G2 a, b; M7G3 a, b, c; M7A1 a; M7A3 a, c, d
Grade 8 – M8A1 a, b, c, e; M8A4 a, b, c, d, f

THE TOWER OF HANOI

The Legend. In an ancient city in India, so the legend goes, monks in a temple have to move a pile of 64 sacred disks from one location to another. The disks are fragile; only one can be carried at a time. A disk may not be placed on top of a smaller, less valuable disk. And, there is only one other location in the temple (besides the original and destination locations) sacred enough that a pile of disks can be placed there. Here's how the game looks with [seven] disks:

So, the monks start moving disks back and forth, between the original pile, the pile at the new location, and the intermediate location, always keeping the piles in order (largest on the bottom, smallest on the top). The legend is that, before the monks make the final move to complete the new pile in the new location, the temple will turn to dust and the world will end. Is there any truth to this legend? (You would need a time reference for moving one disk.)

The Game. There's a game based on this legend. You have a small collection of disks and three piles into which you can put them (in the physical version of this game, you have three posts onto which you can put the disks, which have holes in the center). The disks all start on the leftmost pile, and you want to move them to the rightmost pile, never putting a disk on top of a smaller one. The middle pile is for intermediate storage. How many moves are required? What is the formula?

Web Sites.
Legend from http://www.math.toronto.edu/mathnet/games/towers.html
Cut-out: http://www.lhs.berkeley.edu/java/tower/towerprintout.html
Graphic from interactive site: http://chemeng.p.lodz.pl/zylla/games/hanoi5e.html
Another (similar) legend: http://www.article19.com/shockwave/toh.htm
For more information, search for “Tower of Hanoi” at http://www.google.com/

Georgia Performance Standards
Grade 3 – M3A1 a, c
Grade 4 – M4A1 a, b
Grade 5 – M5A1 a
Grade 6 – M6A2 a; M6A3
Grade 7 – M7A2 a
Grade 8 – M8A1 a, e; M8A3 a, b

Let’s collect some data:

Number of Disks	Moves
1	
2	
3	
4	
n	

How Far Will It Fly?

For making a paper airplane, see the following Web site:
http://www.exploratorium.edu/exploring/paper/airplanes.html

Continue to the next page for further information:
http://www.exploratorium.edu/exploring/paper/airplanes2.html

Nakamura Lock:

1. Fold a sheet of paper in half lengthwise. Unfold so that the crease is 'valley' side up.

2. Fold the top corners down to the center fold.

3. Fold the tip down.

4. Fold about one inch of the tip up; unfold.

5. Fold the top corners down to the center fold so that the corners meet above the fold in the tip. (Note that the top—the nose of the plane—should be blunt.)

6. Fold the tip up. This is the Nakamura lock.

7. Fold the entire plane in half so that the tip is on the outside.

8. Fold the wings down. Trim and fly!

Doherty, P., & Syjuco, S. (n.d.). *Exploratorium, paper airplanes*. Retrieved October 8, 2009, from http://www.exploratorium.edu/exploring/paper/airplanes.html

Procedure:

1. Fly three trials. Record each distance, in inches, from the starting line to where the nose of the airplane lands.
2. Record the median distance on a sticky note, and place it on the continuum on the board.
3. Find the mean, median, and mode for the student trials.
4. Construct a stem and leaf plot.
5. Construct a box and whiskers plot.

Variations

- Use the flying disk toy.

Georgia Performance Standards
Grade 3 – M3M2 b, c; M3D1 a
Grade 4 – M4N4 a, b; M4N7 a; M4D1 a, c, e
Grade 5 – M5D1 a, b, c; M5D2
Grade 6 – M6M2 a, b; M6D1 a, c, d, e
Grade 7 – M7D1 a, c, f, g

Balancing Animals (Equal Shmequal)

Let each animal be represented by the given number of cubes:

Mouse/M	2
Turtle/T	3
Rabbit/R	4
Bobcat/C	5
Wolf/W	8
Deer/D	12
Bear/B	30

Set up the balances:

T = 3 3	<	W = 8 8
T + D = 3 + 12 15	>	W = 8 8
T + D = 3 + 12 15	>	W + C = 8 + 5 13
T + D = 3 + 12 15	<	W + C + R = 8 + 5 + 4 17
T + D + M = 3 + 12 + 2 17	=	W + C + R = 8 + 5 + 4 17
T + D + M = 3 + 12 + 2 17	<	W + C + R + B = 8 + 5 + 4 + 30 47
B + M = 30 + 2 32	=	R + C + W + T + D = 4 + 5 + 8 + 3 + 12 32

Kroll, V., & O'Neill, P. (2005). *Equal shmequal*. Watertown, MA: Charlesbridge.

Original spreadsheet (DM)

Georgia Performance Standards
Grade 3 – M3N2 a, c; M3A1 c
Grade 4 - M4M1 b; M4A1 b, c
Grade 5 - M5A1 a, b
Grade 6 – M6A1; M6A2 b
Grade 7 – M7A1 a; M7A2 a; M7A3 b, c
Grade 8 – M8A1 a, b, e

How to Lift a Lion

Texas Instruments Incorporated. (2000). *Lifting a lion*. Retrieved October 8, 2009, from http://education.ti.com/educationportal/activityexchange/Activity.do?aId=5169

Wells, R. E. (1996). *How do you lift a lion?* Morton Grove, IL: Albert Whitman & Company.

Work = Force * distance
= mass * gravitational acceleration * distance
= mass * 9.81 m/s^2 * distance

Dimensional analysis for work:
= kg * m/s^2 * m
= Newton (N) * m
= Joule (J)

To *balance* the lion, work on each side of the fulcrum must be equal, or $Work_{cl} = Work_{ccl}$. When finding the *mass* at a given distance, gravitational acceleration will cancel from each side of the equation. The TI activity uses the terms work, mass, and weight, which are probably confusing for students in grades 4-6.

Data collection won't be exact since (a) the yardstick will require conversion from inches to cm/m, and (b) the animal's center of gravity might not be placed on the end of the yardstick. One pop cube has a mass of approximately 4 grams.

Worksheet

Find the work required to lift the lion with the fulcrum placed the following distances from the lion:

Yardstick	Ruler	
18 inches	6 (or 6.25) inches	L----------^----------P
27 inches	9 (or 9.375) inches	L----------^------P
9 inches	3 (or 3.125) inches	L------^----------P

Georgia Performance Standards
Grade 3 – M3N3 g; M3A1 c; M3D1 a
Grade 4 - M4N3; M4N4 a, b; M4N7 a; M4M1 a, b; M4A1 a, b, c
Grade 5 - M5A1 a, b
Grade 6 – M6A1; M6A2 a, b, c; M6D1 a, e
Grade 7 – M7A1 a; M7A2 a; M7A3 b, c
Grade 8 – M8A1 a, b, e

Spinning Tops

Let's collect some data:

	Time of Spin (s)		
	Trial 1	Trial 2	Trial 3
Small plastic top			
Large plastic top			
Wooden top			

Variations
- ❖ What other conditions could be varied?
- ❖ Draw a bar graph to represent one set of data.
- ❖ Calculate the mean, median, and mode of the class data.
- ❖ Spin the top on a piece of graph paper. Record starting and ending points, and calculate displacement.

Georgia Performance Standards
Grade 3 – M3M2 b, c; M3D1 a
Grade 4 – M4G3 a, b, c; M4D1 a, c, e
Grade 5 – M5D1 a, b, c; M5D2
Grade 6 – M6M2 a, b; M6D1 a, c, d, e
Grade 7 – M7D1 a, c, f, g

Slope of a Line

Materials: Battery-operated car with two speed settings, meter sticks/tape measures, stopwatches, graph/chart paper, paper and pencil.

Procedure:

- Collect data for each of the two speeds.
- Students will be paired along the track, with one as the timer and one as the recorder. Depending on the speed of the car, place students at every 20, 50, or 100 centimeters.
- Start the car with the front bumper at the zero mark.
- When the car begins to move, all stopwatches should begin.
- When the front bumper of the car reaches each recording point, that stopwatch will be stopped, and the time and distance will be recorded.

- Plot data on a coordinate plane, with time (x-axis) versus distance (y-axis).
- Calculate the rate of the car at each speed setting by finding the slope of each line (D/t).

Georgia Performance Standards
Grade 3 – M3M2 b; M3A1 b, c; M3D1 a
Grade 4 - M4N7 a, b; M4G3 a, b, c; M4A1 a, b, c; M4D1 a, b
Grade 5 – M5N4 a; M5A1 a, b, c
Grade 6 – M6M2 a, b; M6A1; M6A2 a, b; M6D1 a, c, e
Grade 7 – M7A1 a; M7A2 a; M7A3 a, b, c
Grade 8 – M8A1 a, b, e; M8A3 a, b, g, h; M8A4 a, b, c, f; M8D4 a

Making a Pi Necklace
by Diana Funke
Mathematics teacher, Davisville Middle School, North Kingstown, RI

As a 7th grade math teacher, I like to make mathematics as visual as possible. For Pi Day, my students make Pi necklaces.

I use Pi as a way of introducing my students to the idea of an irrational number. After studying decimals that terminate or repeat, I ask them to bring in a can and we compare the circumference to the diameter by dividing C by d. This is how they find out what Pi is all about.

I have them make a Pi necklace to reinforce the idea that some numbers never repeat or end. We usually use from 100 to 300 beads, depending on the size of the bead. They assign a color to each digit (including 0) and then string beads of those colors into a necklace, using the digits of Pi as their guide. Some students make their own beads with polymer clay and others string store-bought beads of all sizes. The first bead, representing the number three, is bigger than the rest. (In the illustration, the necklace has a big silver triangular "bead" as the whole number 3 part of Pi.)

At 1:59 P.M. we all stop what we are doing and wish everyone a Happy Pi Day!

Funke, D., & The Math Forum. (2009). *Making a pi necklace*. Retrieved October 8, 2009, from http://mathforum.org/teachers/middle/activities/pi_day.html

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Added by Deborah McAllister

Pi – to 100 Digits

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Digits from Encyclopedia Uselessia - http://www.greatplay.net/uselessia/articles/pi2-100.html

Key for Beads

Special beads for 3 and decimal point.

Digit	Bead	Number
0	Black	8
1	Pink	8
2	Violet	12
3	Indigo	11
4	Blue	10
5	Green	8
6	Yellow	9
7	Orange	8
8	Red	12
9	White	14

Georgia Performance Standards
Grade 3 – M3A1 a
Grade 4 - M4A1 a
Grade 5 – M5N2 c
Grade 6 – M6D1 a, e
Grade 7 – M7A3 b
Grade 8 – M8A1 e

Solar System Distance Necklace

Challenger Learning Center of Maine Education Committee. (2006). *Solar system distance*. Retrieved October 8, 2009, from http://www.challenger.org/lessons/85.pdf

Each bead different than spacing beads represents the sun, a planet, or a dwarf planet (Pluto). I have modified the chart on the last page to include the sun and additional spacing beads. Note: Each spacing bead represents 30-60 million miles.

Knot the string, leaving some length to tie to the other end. Use elastic string.

Sequence of beads:

Sun – large yellow
1 spacing bead

Inner Planets
Mercury – small yellow
1 spacing bead
Venus – small green
1 spacing bead
Earth – small blue
1 spacing bead
Mars – small red
6 spacing beads

Outer Planets
Jupiter – large orange/red
7 spacing beads
Saturn – large yellow/orange
15 spacing beads
Uranus – medium green
17 spacing beads
Neptune – medium blue/purple
14 spacing beads
Pluto – small black
Spacing beads, as needed for the length of necklace

Knot the end of the string, leaving some length. Tie the ends of the string to form a necklace. The Solar System link has several resources. See http://solarsystem.nasa.gov/planets/

Georgia Performance Standards
Grade 3 – M3N3 a; M3M2 a
Grade 4 – M4N1 a; M4N2 a, b; M4N7 d
Grade 5 – M5D1 a, b
Grade 6 – M6M2 b; M6A1; M6A2 a, b; M6A3; M6D1 a, c, e
Grade 7 – M7G3 b; M7A1 a; M7A2 a; M7A3 b
Grade 8 – M8A1 e; M8A3 a, b