

Origami Solutions for Teaching Selected Topics in Geometry

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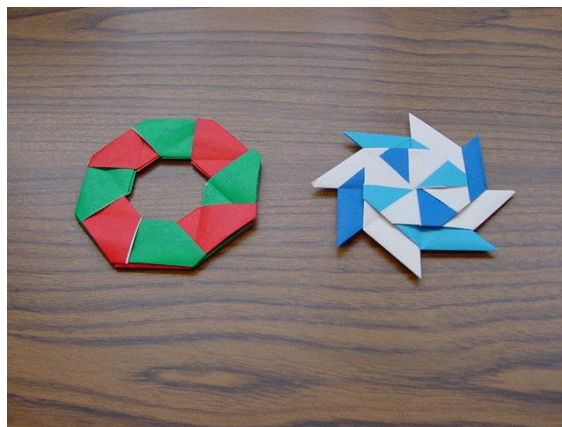
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Session Description

Participants will learn to make origami figures that will be useful for teaching geometry concepts, including reflections, rotations, and geometric solids. Figures to be created will be selected from modular and action origami.

Origami Figures (Grades 3 through 8)

Sailboat
Samurai Helmet
Wreath and Pinwheel
Star
Hexahedron
Cube
Box



Standards References

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author. (See <http://standards.nctm.org/>.)

Tennessee Department of Education. (n.d.). *Curriculum standards, mathematics*. Retrieved from <http://state.tn.us/education/ci/math/index.shtml>

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Sailboat

1. Begin with the colored side of the paper facing up.
2. Orient the paper so that one corner is pointed toward you.
3. Valley fold the bottom corner to the top corner (taco fold). Crease and unfold.
4. Rotate the paper 90 degrees, and valley fold the bottom corner to the top corner. Crease and unfold.
5. Turn the paper over, so that the white side of the paper is facing up.
6. Make two valley folds, bottom edge to top edge and left edge to right edge. Crease and unfold each.
7. Bring the left and right corners of the square together, so that they meet at the bottom corner. The top corner will also meet at the bottom corner.
8. Press flat. This forms the preliminary base.
9. Open the paper, with the white side facing up.
10. Fold the top and bottom corners to the center.
11. As you fold the top and bottom edges together, valley fold the sails to the center.
12. Fold the bottom corner to the center line of the back of the boat. This will form a flat base for the boat.
13. Fold the right-hand sail straight downward.
14. Fold the sail upward, making it shorter than the left-hand sail.
15. Tuck the leftover sail into the base of the boat.
16. Note the similar triangles.

The Sailboat, p. 52-55, from:

Morin, J. (1998). *The ultimate origami book*. Philadelphia, PA: Running Press.

Samurai Helmet

1. Begin with the white side of the paper facing up.
2. Fold the paper in half, diagonally.
3. Place the isosceles triangle on the table with the right angle at the top.
4. Fold each base angle to the right angle.
5. Rotate the square 180 degrees.
6. Fold one of the bottom points to the top corner.
7. To make the horn, fold the top layer of the corner outward, so that the top edge is parallel to the center line.
8. Repeat the previous two steps to make the other horn.
9. Fold a single layer of the paper at the bottom to a point approximately halfway between the top of the helmet and the center line.
10. Fold up the lower edge of the same layer to form a horizontal band.
11. Flatten the helmet.
12. Mountain fold the remaining lower corner to meet the top of the helmet.
13. Tuck the lower corner into the helmet.
14. This piece can be used as a corner bookmark.

Samurai Helmet, p. 49, from:

Beech, R. (2003). *The origami handbook*. London, England: Hermes House.

Wreath and Pinwheel

Materials: Eight paper squares, same size (two colors)

Do the following to each square:

- Fold the square to create four creases: vertical in center, horizontal in center, two diagonals.
- Fold the top corners down to make the roof of a house.
- Fold the house in half such that the flaps are on the inside.
- Hold the half-house by the acute angle at the bottom left.
- Push in the bottom right corner to form a parallelogram.

Making the wreath:

- Position one piece with the folded edge to the left, and the acute angle at the bottom left.
- Position the next piece with the folded edge at the top and the acute angle at the upper left.
- Slide the acute angle on the right piece into the fold pocket of the left piece.
- Fold down the tips of the left piece into the valley of the right piece.
- Attach the remaining pieces.
- Connect the last piece to the first piece.

Making the pinwheel:

Gently slide the sides of the wreath toward the center.

Questions regarding symmetry:

1. Describe the reflectional and rotational symmetries of the following:
 - Square
 - House
 - Half-house
 - Parallelogram
 - Wreath
 - Pinwheel
2. Slide the pinwheel to the wreath. Push on a pair of opposite sides to get a pinwheel with only two wings. What are the reflectional and rotational symmetries of this figure?
3. Slide the pinwheel to the wreath. Push on opposite sides to produce other shapes. What shapes can you create in the center opening?

Making a Wreath and a Pinwheel, p. 73-75, from:

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected mathematics: Kaleidoscopes, hubcaps, and mirrors*. La Porte, IN: Prentice Hall, Dale Seymour Publications.

Star

1. Begin with two squares of coordinating and/or contrasting paper.
2. Book fold, and unfold, each square in half once.
3. For each piece of paper, cut along the crease to form rectangles.
4. You will use one rectangle of each design to form one star.
5. The two rectangles will be folded as mirror images of each other.
6. With the colored side of the paper facing up, fold, and unfold, each rectangle so the short ends meet (hamburger fold).
7. With the white side of the paper facing up, fold each rectangle so the long ends meet (hot dog fold). Crease well.
8. Place each piece on the table with the open end facing away from you, and with one piece above the other.
9. On the upper piece, fold the upper left corner to the crease, forming a flap in the shape of an isosceles right triangle.
10. On the upper piece, fold the bottom right corner to the open edge, forming a flap in the shape of an isosceles right triangle.
11. On the bottom piece, fold the lower left corner to the open edge, forming a flap in the shape of an isosceles right triangle.
12. On the bottom piece, fold the upper right corner to the crease, forming a flap in the shape of an isosceles right triangle.
13. Note the line symmetry (reflection) of the two pieces.
14. Place your fingernail at the top edge of the center crease of the upper piece. Fold the right side downward to form an arrow pointing to the right.
15. Place your fingernail at the bottom edge of the center crease of the lower piece. Fold the right side upward to form an arrow pointing to the right.
16. Place your fingernail at the bottom edge of the center crease of the upper piece. Fold the left side upward to form an arrow pointing to the left.
17. Place your fingernail at the top edge of the center crease of the lower piece. Fold the left side downward to form an arrow pointing to the left.
18. Turn the upper piece over. Do not turn the lower piece.
19. Orient the upper piece so that it looks like a "Z" that has been rotated 45 degrees clockwise. Fold and crease both small triangles (delineated by an open midline on each larger triangle), top downward and bottom upward. Unfold.
20. Orient the lower piece so that it looks like a "Z" that has been rotated 45 degrees clockwise. There are no open midlines on this piece. Fold and crease each triangle into halves, top downward and bottom upward. Unfold.
21. Place the lower piece on top of the upper piece, with the centers aligned, and so that the pieces are perpendicular.
22. Fold the bottom point of the bottom piece up to tuck into the top pocket of the top piece.
23. Fold and tuck the top point of the bottom piece into the bottom pocket of the top piece.
24. Turn the joined pieces over.
25. Tuck the top point of the bottom piece into the left pocket of the top piece.
26. Tuck the bottom point of the bottom piece into the right pocket of the top piece.

Traditional Star, p. 8-12, from:
Gleason, K. (2003). *Christmas origami*. Back Pack Books.

Hexahedron

1. Begin with a square sheet of paper, with the white side of the paper facing upward.
 2. Book fold one vertical crease at center to form halves. Open.
 3. Cupboard fold two vertical creases to center to form fourths. Open.
 4. With the white side of the paper facing upward, fold dog ears at the upper left and the lower right of white side (isosceles right triangles with the corner touching the vertical line at the $\frac{1}{4}$ crease line).
 5. Fold the sides to center (close the cupboard), with the open collar at the upper right and lower left.
 6. Make a horizontal crease at center. Open that crease.
 7. Fold the upper right corner to the mid-point of left-hand side. Open that crease.
 8. Fold lower left corner to the mid-point of right-hand side. Open that crease.
 9. Tuck the upper right corner into the under left-hand flap.
 10. Tuck the lower left corner into the right-hand flap.
 11. Flip the parallelogram so that the smooth side is facing upward.
 12. Fold the acute vertices to the opposite endpoint on the midline (forms a square).
 13. Flip the square so that the rough/tucked side is facing upward.
 14. Fold the piece into a triangle, along the tucked midline.
 15. Open this piece to see four triangles.
 16. A total of three strips are needed (fold two additional strips).
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17. Work with the tucked sides facing upward.
 18. Place one strip vertically.
 19. Place and tuck one strip horizontally at the middle.
 20. Place and tuck one strip vertically at the center.
 21. Push the unit up into a three-sided pyramid.
 22. Hold the unit together as three-sided pyramid.
 23. Tuck the loose ends into the pockets to form a hexahedron.

Cube

1. Begin with a square sheet of paper, with the white side of the paper facing upward.
 2. Book fold one vertical crease at center to form halves. Open.
 3. Cupboard fold two vertical creases to center to form fourths. Open.
 4. With the white side of the paper facing upward, fold dog ears at the upper left and the lower right of white side (isosceles right triangles with the corner touching the vertical line at the $\frac{1}{4}$ crease line).
 5. Fold the sides to center (close the cupboard), with the open collar at the upper right and lower left.
 6. Make a horizontal crease at center. Open that crease.
 7. Fold the upper right corner to the mid-point of left-hand side. Open that crease.
 8. Fold lower left corner to the mid-point of right-hand side. Open that crease.
 9. Tuck the upper right corner into the under left-hand flap.
 10. Tuck the lower left corner into the right-hand flap.
 11. Flip the parallelogram so that the smooth side is facing upward.
 12. Fold the acute vertices to the opposite endpoint on the midline (forms a square).
 13. Flip the square so that the rough/tucked side is facing upward.
 14. Fold the piece into a triangle, along the tucked midline.
 15. Open this piece to see four triangles.
 16. A total of six strips are needed (fold five additional strips).
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17. Work with the tucked sides facing upward.
 18. Place one strip vertically.
 19. Place and tuck one strip horizontally at the middle.
 20. Place and tuck one strip vertically at the center.
 21. Push the left flap under the uppermost piece to form a corner of the cube.
 22. Hold the unit together as a corner.
 23. Tuck additional pieces into the pockets to form a cube. Each face of the cube will contain three colors, with one color on one pair of opposite sides, and two different colors on the other pair of opposite sides.

Box

1. Begin with the colored side of the paper facing up.
2. Make a book fold, and unfold, in both directions.
3. Turn the paper to the white side facing up.
4. Fold each corner to the center.
5. Cupboard fold, and unfold, the top edge and the bottom edge to the center.
6. Cupboard fold, and unfold, the left edge and the right edge to the center.
7. Unfold the top and bottom triangular flaps.
8. Turn up and crease the left and right cupboard folds to 90 degrees to form two sides of the box.
9. Push inward on the small mountain-fold creases where the colored and white sides of the paper meet. Make a valley fold on the crease that connects these two mountain folds. This will form the third side of the box.
10. Pull the tab down and inside the box to line the box.
11. Repeat the previous two steps to form the fourth side of the box.
12. Reinforce the creases on the box.
13. To make the box cover, repeat the above directions, leaving a small gap at the center when making the cupboard folds.

Masu Box, p. 56-57, from:

Gross, G. M. (2001). *Origami, Easy to make paper creations*. New York, NY: Michael Friedman Publishing Group, Inc.

NCTM Standards

Standard 3 Geometry

Goal: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Grades 3-5 Expectations:

- identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;
- classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;
- investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;
- explore congruence and similarity.

Grades 6-8 Expectations:

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Goal: Apply transformations and use symmetry to analyze mathematical situations.

Grades 3-5 Expectations:

- predict and describe the results of sliding, flipping, and turning two-dimensional shapes;

- identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.

Grades 6-8 Expectations:

- describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Goal: Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grades 3-5 Expectations:

- build and draw geometric objects;
- use geometric models to solve problems in other areas of mathematics, such as number and measurement;
- recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

Grades 6-8 Expectations:

- use geometric models to represent and explain numerical and algebraic relationships;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Standard 8 Communication

Goals:

Organize and consolidate their mathematical thinking through communication.
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
Analyze and evaluate the mathematical thinking and strategies of others.
Use the language of mathematics to express mathematical ideas precisely.

Standard 9 Connections

Goals:

Recognize and use connections among mathematical ideas.
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
Recognize and apply mathematics in contexts outside of mathematics.

Standard 10 Representation

Goals:

Create and use representations to organize, record, and communicate mathematical ideas.
Select, apply, and translate among mathematical representations to solve problems.
Use representations to model and interpret physical, social, and mathematical phenomena.